NAME _____

Follow the directions carefully. Your grade will be based on the **quality and depth** of your responses in this timed environment. The back of the last page is blank if you need more room. Please note that informal phrasing and bullet points are fine. GOOD LUCK!! When you are finished, turn in your cheat sheet along with your test and have a nice weekend. Homework is online for Monday.

Problem 1: In *Heaven and Hell,* from 1960*,* M.C. Escher has created creatures "living" in a 2-dimensional space, which may or may not be flat, onto the page, which is definitely flat. The creatures are **all the same size** in their own world.

Part A In the picture below, I have labeled three points. Calculate the sum of the angles in the triangle formed by these and then use the sum to determine whether this space is Euclidean, spherical or hyperbolic. **Show your calculation work too**.
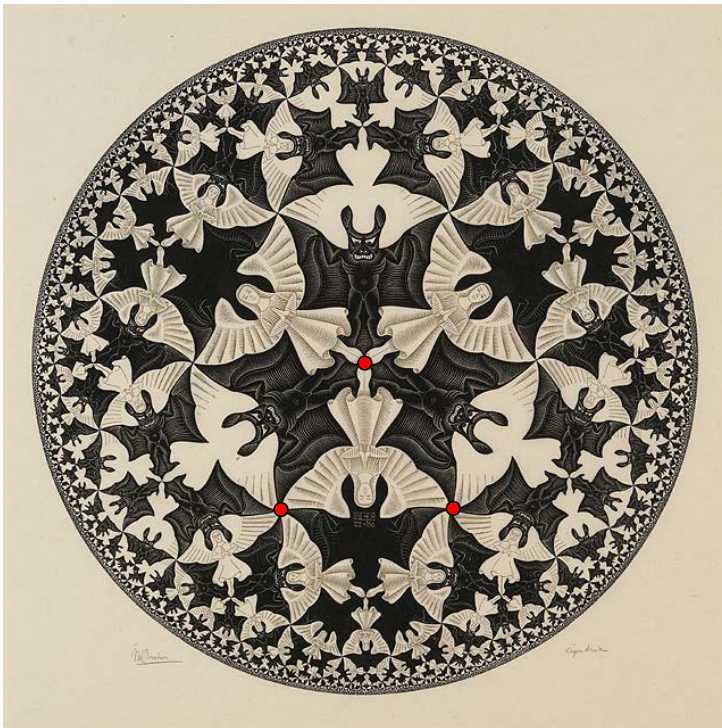
Angle 1 calculation:

Angle 2 calculation:

Angle 3 calculation:

Sum of the angles:

Euclidean, spherical, or hyperbolic space?

Part B Are there any intrinsically straight parallels (ie from the perspective of someone inside the space – parallels that feel straight) in the geometry you selected? Briefly explain why or why not.

Problem 2: In class and lab we discussed what life would be like for Arthur Square or other 2D creatures from *Flatland*.

Part A How could Arthur Square pass another creature in *Flatland*?

<u>Part B</u> Now imagine a **sphere** passing through the plane. **Sketch a few pictures** that reflect what **Arthur Square would actually see in the 2-D universe** at different times of the dunking (assume that he can only see in the 2-D universe).
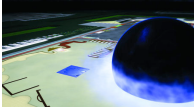
Problem 3:
John Playfair's (1748-1819) postulated that there is only one parallel to a line or symmetric path through a given point. Is this true on the Euclidean plane, in hyperbolic geometry, and/or the sphere? Briefly explain what happens in each geometry.

Problem 4:
<u>Part A</u> What is a probability and how does it relate to truth?

<u>Part B</u> Analyze the role of probability and chance in computing the density and geometry of the universe.

<u>Part C</u> Discuss an instance from the geometry segment where the theme of local to global played a role. Be sure to specify what was local, what was global, and how these perspectives related or differed.

Problem 5: Recall that one possible Euclidean 2-D Universe is a Klein bottle. In Klein bottle tic-tac-toe, we identify the left and right sides of the board straight across. The top and bottom are glued via a **reflection** in the vertical line through the middle of the game board. This would then form the Klein bottle.

Part A Draw a tiling view of the original game pieces, above, below, and to the right and left of the main game board.

Part B I am "X" in the game and I can win Klein bottle tic-tac-toe with my next move. Mark off where I can go to win on the main board
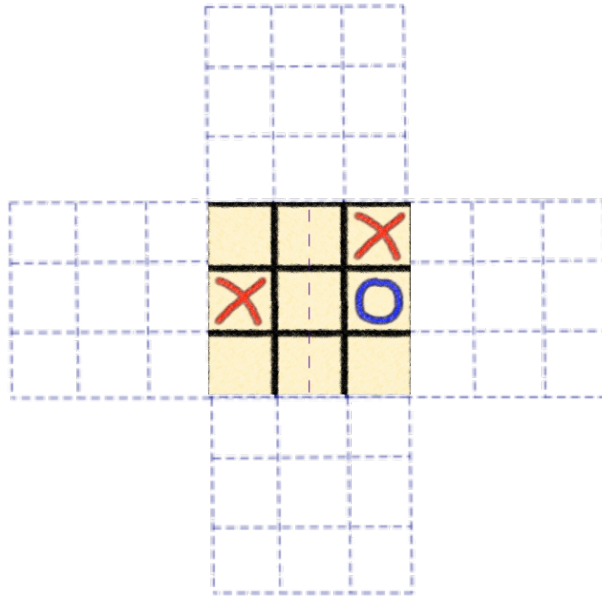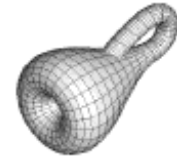
Problem 6:

Part A Choose a method researchers have employed to determine whether our universe satisfies the laws of Euclidean, spherical, or hyperbolic geometry and describe it briefly.

Part B Discuss at least 2 of our **classroom critiques** of this method.