

## The Use of Letter Writing Projects in Teaching Geometry

Sarah J. Greenwald

ADDRESS: Department of Mathematics, Appalachian State University, Boone, NC 28608

ABSTRACT: Assignments in which students write letters to a non-mathematician who needs their help enable students to explore mathematical ideas deeply and encourage them to communicate mathematics clearly while providing them with the benefits of creative writing in a familiar format aimed at a non-expert audience. Teachers who recognize the benefits of exploratory writing in mathematics are often hesitant to utilize this method because of the difficulties and time involved in grading papers. This paper will provide examples of geometry writing projects and will discuss the goals and difficulties of using these projects. The process of grading and allowing student revisions of these projects will be explored by using a time-saving grading method.

KEYWORDS: Writing, geometry, exploration, grading, revisions

In letter writing projects, students write letters to Wile E. Coyote [9] or another non-mathematician who needs their help. Such assignments enable students to explore mathematical ideas deeply and encourage students to communicate mathematics clearly while providing them with the benefits of creative writing in a familiar format aimed at a non-expert audience.

I first encountered this type of writing project on Dr. Tommy Ratliff's "Writing in Calculus" web page [6] and I have since adapted the assignments for use in teaching geometry. After giving an example of a geometry writing project, I will explore the grading of student responses using a checklist based on one adapted from Dr. Annalisa Crannell's "Writing in Mathematics" web page [2]. I will also discuss the benefits and difficulties of using writing projects in teaching mathematics.

The following letter is an example of a possible first assignment for students in a geometry course. The purpose of the assignment is to introduce students to exploratory learning, so I would give this out early in the semester, before the class has discussed spherical geometry. The students are forced to develop creative responses instead of reiterating classroom discussions, because the students have not previously encountered the material.

#### **First Assignment**

Dear Math 3610 Students:

I keep having a recurring nightmare. I'm trapped on the surface of a ball chasing the roadrunner, when all of a sudden he disappears from my view. We are both trapped on the surface, so I know he can't have gone too far, right? All I can think about is catching that roadrunner. I'm no mathematician, so please tell me what to do! If I keep running in the same direction, will I be able to see him? Will I then be able to catch him? Am I better off standing still? Or perhaps I should start changing directions?

Your professor will need your report in her box by 5 P.M. on Monday, so that she can bring it to me.

Help me -- you're my only hope!

Hungry as ever,

Wile E. Coyote

P.S. Are there any Acme gadgets which would be helpful?

Students are typically amused when they receive this assignment, but they are also concerned about the expectations and the letter writing. Their concerns are understandable since this assignment is meant to alter the way they learn and communicate mathematics. I give the following checklist to students along with the assignment in order to provide some guidance.

#### **Blank Checklist Handed Out Along With Geometry Writing Projects**

Adapted from a checklist by Crannell [1]

##### **Does this paper...**

- clearly (re)state the problem to be solved?
- provide a paragraph which explains how the problem(s) will be approached?
- state the answer(s) in a few complete sentences which stand on their own?
- give a precise and well-organized explanation of how the answer was found?
- clearly summarize diagrams, graphs, or other visual representations of math?
- define all variables, terminology and notation used?
- clearly state the underlying assumptions and explain how each is derived?
- give acknowledgment where it is due?
- use correct spelling, grammar and punctuation?
- contain correct mathematics?
- solve all of the questions that were originally asked?
- give evidence of creative writing?
- give evidence of deep mathematical exploration of the problem(s)?

I added the final two checklist points involving creative and deep exploration to Crannell's original checklist [1] in order to distinguish top student papers and to encourage exploration. When handing out the checklist with the assignment, I instruct students to ignore points on the list that are not relevant to this assignment or to their specific paper. For example, the checklist point on providing a paragraph explaining how the problem will be approached may not be applicable in this assignment because students are not working on an extensively complex mathematical problem that needs this type of introduction. As part of the assignment, students must figure out which checklist points are relevant

for their papers. I also clarify that students should deeply explore the underlying mathematics involved and then communicate their mathematical ideas and explanations in a well organized, creative letter. Students understand that both creative exploration and deep mathematical exploration of ideas are important and that they cannot receive a perfect score unless they have accomplished both of these explorations. In order to further alleviate student concerns about the expectations of the assignment, I explain my system of allowing revisions and encourage students to bring drafts into office hours. After I return their graded papers, the students typically have one week to revise papers by responding to my comments. The revised grade replaces the original grade, with the stated understanding that students who consistently turn in high quality work as a first response will receive extra credit.

The following is a student's actual first response, including spelling and grammatical errors. As a first step in grading, I have underlined statements needing correction or more explanation. I would also place comments in the margin which encourage exploration, such as "clarify," "why" or "elaborate," in addition to positive feedback. Instead of appearing in the margins, my comments in this paper appear bolded and contained within brackets.

#### **Student I's First Response with My Comments**

Dear Wile E. Coyote,

Your strategy for catching the roadrunner would seem to be contingent on the path and speed of the roadrunner. If you are chasing the roadrunner and he is disappearing from sight because his velocity is greater than yours, you're [reword] best strategy might be to turn completely around (180 degrees) and begin running. You should eventually run into him head on [why?]. Since the sphere is obviously small enough that he is disappearing from your view, it should not take too long for you to run into him again [great but clarify]. This about-face strategy does require that the roadrunner stay on the same course [explain]. If he greatly deviates from the course you last saw him on he will pass by you, possibly out of your sight [nice but clarify]. Another possible strategy would be to turn 135 degrees [which way?] and begin running. This strategy offers no guarantee that you will catch him again, but if you do you should blindside him from the back [reword] and have a delicious dinner. As far as the acme gadgets are concerned[,], the explosive prima-cord could be laid on a path perpendicular to your present course

and exploded[, **nice!**] thereby cutting the sphere in half. From that point, the roadrunner will run into you [**why?**] if you happen to have been lucky enough that he is in the same hemisphere as you are.

Good Hunting,

Student's Name

Before grading, I try to decide on realistic mathematical expectations for the class, given that I have not covered the material. For this assignment, I expected that students would discuss reasonable paths on the surface of the sphere and explain why they seem reasonable. While students are not expected to understand that a longitudinal line is the shortest path on a sphere, they can still explore the concept of paths by thinking about water droplets on the surface or airplane flight patterns on the earth. I also expect students to state and explain their underlying assumptions and explain their claims. This student's response is a good first effort, but the student needs to elaborate and explain the mathematics underlying many of the statements in order to receive a perfect score.

Students often view the grading of written assignments as a mysterious process. In order to ensure fair grading and to communicate the process to the student, I use the checklist I gave out with the assignment. While grading is time consuming at first, the checklist ensures fair grading and reduces grading time once the grader is familiar with the process. To shorten grading time, I look for specific ideas based on my expectations while first reading the paper, add comments in the margin, look back at the paper and then check off points on the checklist. For example, in this assignment, I was specifically looking for the student's definition of a path and an explanation after every statement or claim. I place a check after points that the student has covered well, a minus sign after points that need work, and I leave blank the points that are not relevant to the specific assignment. I also emphasize ideas by writing select comments on the checklist, seen here in bold.

#### **Graded Checklist for Student I's First Response**

Adapted from a checklist by Crannell [1]

**Does this paper...**

- clearly (re)state the problem to be solved? --
- provide a paragraph which explains how the problem(s) will be approached?
- state the answer(s) in a few complete sentences which stand on their own? --
- give a precise and well-organized explanation of how the answer was found?
- clearly summarize diagrams, graphs, or other visual representations of math?
- define all variables, terminology and notation used?
- clearly state the underlying assumptions and explain how each is derived? --
- give acknowledgment where it is due?
- use correct spelling, grammar and punctuation? -- **mostly**
- contain correct mathematics? -- **Elaborate on the math behind your statements.**
- solve all of the questions that were originally asked? ✓
- give evidence of creative exploration of the problem(s)? ✓ **great!**
- give evidence of deep mathematical exploration of the problem(s)? -- **You could explore paths on the ball or something else mathematical.**

After filling out the checklist, I carefully examine both the checklist and the original paper with my comments, and then I assign a grade to the student. This paper was given 6/10, and the student was told that he had good, creative ideas but needed to elaborate on the mathematics involved. Students have approximately one week to turn in optional revisions after receiving their graded papers and are directed to turn in the revision, the original paper and my marked checklist. I do not require a second draft, because in my experience students appreciate revisions when they see them as a chance to improve their grade, but they resent the extra work when revisions are required.

I find that allowing revisions actually shortens the grading process. In grading the original paper, I spend my grading time on comments designed to encourage revision instead of worrying about correcting every mistake and providing solutions. Grading becomes a positive process meant to help students turn in higher quality revised papers, while at the same time highlighting points they have

already done well, instead of a process concentrating on errors. I do correct mistakes in revised papers. However, since most students correct the mistakes from their first drafts on their own, I end up having very few revised papers with ideas needing major correction. While these few papers are time consuming, I would have still spent this time grading if I had not allowed revisions. For the other students, revised papers are easy to grade with the original paper, comments and checklist in hand. Hence, grading takes less time overall. For a typical example of the complete process, see Appendix B.

One of the difficulties of using creative letter writing projects is that students may ignore the assignment and instead concentrate on creative ideas.

#### Student II's First Response with My Comments

Dear Mr. Coyote,

I just got your letter in the mail, and I can't express how upset with you I am [**reword**]. For over twenty years now, you have been chasing that poor roadrunner, and it's about time someone put an end to it. Don't you realize that no matter how hard you try, you will never catch the roadrunner, [?] How many trips down the side of a cliff via gravity does it take you to realize that the roadrunner is just too good for you? How many sticks of dynamite must go off in your own hands before you give up? [**creative**] If I were you, Mr. Coyote, I would give up chasing roadrunner, and try for something more my speed, like a one-legged cow. Please, before it's too late, [**too late for what?**] leave the poor roadrunner alone.

Your friend,

Student's Name

The above student received 1/10 for creativity, with a "minus" placed next to the other relevant checklist points. The student turned in a revised paper that was much improved.

Another difficulty is that students may concentrate on creative ideas that are mathematically invalid since the projects are often based upon subjects that we have not already covered in class. For example, in the *Homer Simpson Changes Dimensions Writing Project* (see Appendix A), students have incorrectly assumed that Homer has gained the ability to fly because of his change in physical dimensions. Since these projects are designed to stimulate both mathematical exploration and creative

writing, I respond with the comment “I don’t understand” instead of a comment that discourages the creative process. I purposely do not correct the paper in order to encourage students to correct mistakes themselves for the revision. I rarely directly tell a student that ideas are false, since I want them to explore their own ideas instead of trying to guess at which answer I might expect. In addition, concepts that seem invalid at first are often fine once the student has refined, explained and clarified the ideas.

Letter writing projects encourage students to develop creative writing skills and to communicate mathematics to a general audience. Students learn that the ability to deal with new concepts and pose well-formed questions and conjectures is as much a part of mathematics as solving equations is. In addition, as a result of the revision process, students see the learning of mathematics as a dynamic process that extends past the due date of an assignment, as seen in final course evaluations.

Letter writing projects are especially effective in geometry since they enable students to identify personally with geometric situations. For example, instead of theoretical proofs about spherical geometry, students imagine themselves and Wile E. Coyote obeying the laws of spherical geometry. Or, instead of seeing an axiom system as abstract, students see that the axioms and subsequent proofs affect the life of the person trapped in the system, as in the *Wile E. Coyote’s Axiom System Nightmare Writing Project* (see Appendix A). The revision process encourages students to think deeply about both the writing and the mathematics in the first drafts, which often leads to self-correction (see Appendix B for an example of the complete writing and grading process). As one student wrote in her revision: “I had to really think about what the axioms meant and how they related to the chase.” This method does not take the place of standard proofs, problem sets and tests. Instead, supplementing standard techniques with writing projects enables students to develop geometric intuition and creative writing skills along with rigorous proof techniques.

## APPENDIX A: SAMPLE WRITING PROJECTS

### **Homer Simpson Changes Dimensions Writing Project**

In the 3-D Homer segment from *The Simpsons*' "Treehouse of Horror VI" [7], Homer Simpson goes from 2-D into 3-D. We will watch this in class along with the Geometry Center video "The Shape of Space" [3]. In lab, we will watch web video and listen to web audio from this episode and we will use the web to explore some of the issues Pacific Data Images faced while creating the episode.

1) Write a letter from Homer Simpson to his family which discusses his change from 2-D into 3-D. Be sure to include mathematical explanations in Homer's words (as opposed to Professor Frink's words), in addition to physical descriptions.

2) Pretend that Homer Simpson and family had been 3-D all along and that he changed into 4-D (length, width, height and some other physical dimension (not time) -- call it  $w$ ). In many respects, *The Simpsons* represent a 3-D world. For example, when the characters pass one another, they don't jump over each other as the 2-D creatures do in "The Shape of Space" video. Write a letter from Homer to his family discussing his jump from 3-D into 4-D.

3) Compare and contrast your letters.

### **Wile E. Coyote's Axiom System Nightmare Writing Project**

Dear Math 3610 Students:

I keep having a recurring nightmare where I am trapped on the in the following axiom system:

A1: Coyotes and roadrunners live on the planet earth.

A2: Roadrunners run 10 mph faster than the maximum speed of coyotes.

A3: Coyotes always begin chasing roadrunners exactly 2 seconds after roadrunners come into view.

A4: Coyotes only stop chasing roadrunners when they disappear from view.

A5: All coyotes have 20/20 vision.

Can I catch the roadrunner in this axiom system? Can we add another axiom to the system which is consistent with A1 through A5 and ensures that I will always catch the roadrunner? Please prove your claims to me!

Your professor will need your report by 5 P.M. on Monday, so that she can bring it to me.

Help me -- you're my only hope!

Hungry as ever,

Wile E. Coyote

APPENDIX B: SAMPLE OF THE ENTIRE PROCESS FOR *WILE E. COYOTES AXIOM SYSTEM*

*NIGHTMARE WRITING PROJECT* (see Appendix A)

**Student III's First Response with My Comments**

Dear Wile,

I heard you had another recurring nightmare where you are trapped in the following axiom system:

A1: Coyotes and roadrunners live on the planet earth.

A2: Roadrunners run 10 mph faster than the maximum speed of coyotes.

A3: Coyotes always begin chasing roadrunners exactly 2 seconds after roadrunners come into view.

A4: Coyotes only stop chasing roadrunners when they disappear from view.

A5: All coyotes have 20/20 vision.

Your question is, Can you catch the roadrunner on this axiom system? My answer is no for the following reasons. First of all, Because of A3 you are always going to be behind the roadrunner [**not necessarily**]. A2 makes you way behind him because he's most likely going to be running at top speed when you start running from a stop. Even when you are at top speed, he is putting more distance between you [**elaborate**]. Pretty soon the roadrunner will disappear from view since coyotes have 20/20 vision and you will naturally stop chasing him because of A4. Therefore, there is no hope that you will catch the roadrunner on this system.

However, you shouldn't give up hope. I believe we can add some axioms to the system that will ensure that you catch those pesky roadrunners every time. The axioms I came up with are as follows:

A6: Coyotes always follow the exact same path as roadrunners.

A7: Roadrunners always run past coyotes in order to tease them.

A8: Coyotes wearing roller skates on their feet and jet propellers on their backs can travel 20 mph faster than roadrunners. [**nice axioms!**]

Let me prove to you that this will work. First of all, A6 ensures that you will always be right behind the roadrunner in your dream. With this assurance the roadrunner won't be able to veer off the path and out of your sight in order to lose you. A7 means that at some point in time the roadrunner will cross your path [**good!**]. Thus, you will see him and begin the chase. When you order and wear your handy-dandy Acme roller skates and jet propeller, you are guaranteed to catch him. Even if you are starting from a dead stop, your speed will allow you to keep the roadrunner within your line of sight. Your increased speed will also allow you to slowly gain on the roadrunner until he is within your grasp. You might consider having your fork and knife ready because you will finally get to sink your teeth into that roadrunner you've always dreamed of.

I wish you happy hunting and sweet dreams.

Talk to you later,

Student's Name

### Graded Checklist for Student III's First Response

Adapted from a checklist by Crannell [1]

#### Does this paper...

- clearly (re)state the problem to be solved? ✓ **nice!**
- provide a paragraph which explains how the problem(s) will be approached?
- state the answer(s) in a few complete sentences which stand on their own? ✓
- give a precise and well-organized explanation of how the answer was found? **--redo proofs**
- clearly summarize diagrams, graphs, or other visual representations of math?
- define all variables, terminology and notation used?
- clearly state the underlying assumptions and explain how each is derived? ✓
- give acknowledgment where it is due?
- use correct spelling, grammar and punctuation? ✓ **mostly**
- contain correct mathematics? **--clarify and add rigor to proofs, careful with "behind"**
- solve all of the questions that were originally asked? **--consistency of axioms?**
- give evidence of creative exploration of the problem(s)? ✓ **very nice!**
- give evidence of deep mathematical exploration of the problem(s)? --

**Nice start! 7/10**

### Student III's Second Response with My Comments

Dear Wile,

I heard you had another recurring nightmare where you are trapped in the following axiom system:

A1: Coyotes and roadrunners live on the planet earth.

A2: Roadrunners run 10 mph faster than the maximum speed of coyotes.

A3: Coyotes always begin chasing roadrunners exactly 2 seconds after roadrunners come into view.

A4: Coyotes only stop chasing roadrunners when they disappear from view.

A5: All coyotes have 20/20 vision.

Your question is, can you catch the roadrunner on this axiom system? My answer is no for the following reasons. First of all, because of A3 you are always going to be starting out slower than the roadrunner and 2 seconds behind him. I say behind because he is going to be somewhere ahead of you in relation to your starting point. For example, if you are standing on the 0 of a number line, the roadrunner is going to come from the negative direction and be at some point in the positive direction after he passes you, and you start running. A2 ensures that you are way behind him because he is going 10 mph faster than your maximum speed when you start running from a stop. Thus, when you are at top speed, he is putting more distance between the two of you. Consequently, the roadrunner will disappear from view, since coyotes have 20/20 vision. Then you will naturally stop chasing him because of A4. Therefore, there is no hope that you will catch the roadrunner in this system. **[Nice revision for this part, but what happens if the roadrunner never comes into view? It might be better to introduce your proof with the following: "If the coyote can see the roadrunner then the coyote is chasing him or will begin chasing him within 2 seconds." Conclude with: "Otherwise the coyote can't see the roadrunner so he has no hope of catching him."]**

However, you shouldn't give up hope. I believe we can add some axioms to the system that will ensure that you catch those pesky roadrunners every time. The axioms I came up with are as follows:

A6: Coyotes always follow the exact same path as roadrunners.

A7: Roadrunners always run past coyotes in order to tease them.

A8: Coyotes wearing roller skates on their feet and jet propellers on their backs can travel 20 mph faster than roadrunners.

Let me prove to you that this will work. First of all, axioms 6 through 8 do not conflict with the other axioms, so this new system is consistent [**good**]. A6 ensures that you will always be following the roadrunner in your dream. By following, I mean that you always travel the same path as the roadrunner. If he turns off to the right or left, you will do the same. Thus, the roadrunner will not be able to veer off the path and out of your sight in order to lose you [**nice!**]. A7 means that at some point in time the roadrunner will cross your path. Consequently, you will see him and begin the chase by A3. Provided you order and wear your handy-dandy Acme roller skates and jet propeller, you are guaranteed to catch the pesky roadrunner. Even if you are starting from a dead stop, your speed will allow you to keep the roadrunner within your line of sight.

This aid will also allow you to slowly shorten the gap between yourself and the roadrunner until he is within your grasp. Therefore you will be certain to catch the roadrunner at last and your nightmares will finally be over. Note: you might consider having your fork and knife ready because you will finally get to sink your teeth into that roadrunner you've always dreamed of.

I wish you happy hunting and sweet dreams.

Best wishes,

Student's Name

[**very nice revision! 9/10**]

## REFERENCES

- <sup>1</sup> Crannell, Annalisa, How to Grade 300 Mathematical Essays and Survive to tell the Tale, PRIMUS, Vol. 4, No. 3, 1994, 193-201.
- <sup>2</sup> Crannell, Annalisa, "Writing in Mathematics" web site,  
[http://www.fandm.edu/Departments/Mathematics/writing\\_in\\_math/writing\\_index.html](http://www.fandm.edu/Departments/Mathematics/writing_in_math/writing_index.html).
- <sup>3</sup> Geometry Center, "Geometry Center Video Productions" web site,  
<http://www.geom.umn.edu/video/>.
- <sup>4</sup> Meier, John and Rishel, Thomas, Writing in the Teaching and Learning of Mathematics, MAA Notes Series, No. 48, MAA, Washington D.C., 1998
- <sup>5</sup> Ratliff, Tommy, Making Group Projects in Calculus Manageable and Creative, UME Trends, Vol. 7, No. 4, September, 1995, 15-16.
- <sup>6</sup> Ratliff, Tommy, "Writing in Calculus" web site,  
<http://www2.wheatonma.edu/academic/academicdept/MathCS/faculty/tratliff/writing>
- <sup>7</sup> *The Simpsons* , TM and copyright Fox and its related companies, "Treehouse of Horror VI," episode 3F04, original airdate 10/30/95.
- <sup>8</sup> Sterrett, Andrew [Ed.], Using Writing to Teach Mathematics, MAA Notes Series, No. 16, MAA, Washington D.C., 1992
- <sup>9</sup> *Wile E Coyote*, copyright Warner Brothers.

### **Biographical Sketch**

Sarah J. Greenwald received her PhD in 1998 from the University of Pennsylvania and received her BS from Union College in Schenectady NY in 1991. She is currently an assistant professor of mathematics at Appalachian State University in Boone NC, and a research associate at Dartmouth College. Her research area is in the Riemannian geometry of orbifolds, and she is also interested in exploration-based teaching using computers and writing projects. She is currently creating and teaching Women in Mathematics, a course with portions dedicated to both mathematics content and equity issues (see <http://www.mathsci.appstate.edu/~sjg/womeninmath>).