Towards a Module System for K*

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Abstract. Research on the semantics of programming languages has yielded a wide array of notations and methodologies for defining languages and language features. An important feature many of these notations and methodologies lack is *modularity*: the ability to define a language feature once, insulating it from unrelated changes in other parts of the language, and allowing it to be reused in other language definitions. This paper introduces ongoing work on modularity features in K, an algebraic, rewriting logic based formalism for defining language semantics.

Key words: language semantics, rewriting logic, modularity, K

1 Introduction

One important aspect of formalisms for defining the semantics of programming languages is modularity. Modularity is generally expressed as the ability to add new language features, or modify existing features, without having to modify unrelated semantic rules. For instance, when designing a simple expression language, one may want to use structural operational semantics (SOS) [29] to define the semantics of addition:

$$\frac{e_1 \rightarrow e_1'}{e_1 + e_2 \rightarrow e_1' + e_2} \qquad \text{(EXP-PLUS-L)}$$

$$\frac{e_2 \rightarrow e_2'}{n_1 + e_2 \rightarrow n_1 + e_2'} \qquad \text{(EXP-PLUS-R)}$$

$$n_1 + n_2 \rightarrow n, \text{where } n = n_1 + n_2 \qquad \text{(EXP-PLUS)}$$

Further extending the language, one may want to add variables. The standard way to do this is to define a store, mapping names to values, with rules for binding values to names (not shown here) and to retrieve the current value of a binding:

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$$\langle x, \sigma \rangle \rightarrow \langle n, \sigma \rangle, \text{where } n = \sigma(x) \qquad \text{(VAR-LOOKUP)}$$

With this change to the language, even though the rules for plus do not actually reference the store they must still be modified to include it as part of the configuration. As an example, rule `EXP-PLUS-L` becomes[1]:

$$\frac{\langle e_1, \sigma \rangle \rightarrow \langle e_1', \sigma \rangle}{\langle e_1 + e_2, \sigma \rangle \rightarrow \langle e_1' + e_2, \sigma \rangle} \qquad \text{(EXP-PLUS-L)}$$

Similar types of changes to existing rules need to be made to accommodate other unrelated language features, such as exceptions or function returns. Alternatively, similar changes may need to be made to add addition expressions to a different language with a different configuration, even if the different elements of the configuration are not used in the rules for addition. All these changes are required because SOS is not modular. Improved support for modularity eliminates the need to make these changes, offering several advantages:

- Modular definitions of language features allow other parts of a language to change more easily by allowing existing feature definitions to remain unchanged in the face of unrelated modifications or additions;
- A modular definition of a language feature can be more easily reused in the definition of a different language which may be structured much differently;
- Modular definitions are easier to understand, since the rules given for a language construct only need to include the information needed by the rule, instead of including extraneous information used in other parts of the language (such as the store in the rules for plus).

For these reasons, improving modularity of language definitions has been a focus of research across multiple semantic formalisms. One example is modular structural operational semantics (MSOS) [25, 26], which solves the problem shown above by leveraging the labels on rule transitions, not normally used in SOS definitions of programming languages, to encode configuration elements, with the ability to elide unused parts of the configuration. This is discussed further with other related work in Section 4.

With a tool supported semantics, modularity can also be expressed as the ability to package language features into discreet reusable units, which can then be assembled when defining a language. This form of modularity depends on the first: it should be possible to plug the same feature into multiple definitions, even in cases where (unused) parts of the configuration are different. Additionally, it should be possible to provide clean interfaces to language features and to different parts of the configuration, something not required in monolithic definitions, or even in modular definitions written on paper.

[1] A more general version of this rule would use σ on the left and σ' on the right; here, by using σ on both left and right, we state that expressions do not alter the store, i.e. they do not have side effects.

This paper provides a high-level overview of ongoing work on adding modularity features to K [30], an algebraic, rewriting logic based formalism for programming language semantics. This work is focused on both aspects of modularity mentioned above, allowing the packaging of language features for reuse while insulating existing features from unrelated changes to the language definition. Novel aspects of the module system include the use of context transformers, described along with K in Section 2; the incorporation of some features, such as the explicit hiding or requiring of operations and sorts, which are common in the module systems of programming languages but do not appear to be common in systems for modular language definition; and the methods used to assemble the final language and define the shape of the configuration, chosen to support having multiple distinct semantics for a language and with an eye towards future improved tool support, including visualization.

The remainder of this paper is organized as follows. First, we provide a brief overview of term rewriting, equational logic, rewriting logic, and especially K in Section 2. Next, Section 3 introduces the module system through fragments of a simple imperative language and illustrates the ability to reuse modules in language extensions. Section 4 then reviews related work, while in Section 5 we conclude and discuss future work.

2 Rewriting Logic

This section provides a brief introduction to term rewriting, rewriting logic, rewriting logic semantics, and K. Term rewriting is a standard computational model supported by many systems; rewriting logic [19, 18] organizes term rewriting modulo equations as a complete logic and serves as a foundation for programming language semantics using rewriting logic semantics [21, 22]. K [30] is a rewrite-based method for formally defining computation, here used to provide formal definitions for programming languages.

2.1 Term Rewriting

Term rewriting is a method of computation that works by progressively changing (rewriting) a term. This rewriting process is defined by a number of rules – potentially containing variables – which are each of the form: $l \rightarrow r$. A rule can apply to the entire term being rewritten or to a subterm of the term. First, a match within the current term is found. This is done by finding a substitution, θ, from variables to terms such that the left-hand side of the rule, l, matches part or all of the current term when the variables in l are replaced according to the substitution. The matched subterm is then replaced by the result of applying the substitution to the right-hand side of the rule, r. Thus, the part of the current term matching $\theta(l)$ is replaced by $\theta(r)$. The rewriting process continues as long as it is possible to find a subterm, rule, and substitution such that $\theta(l)$ matches the subterm. When no matching subterms are found, the rewriting process terminates, with the final term being the result of the computation.

Rewriting, like other methods of computation, can continue forever. There exist a plethora of term rewriting engines, including ASF+SDF [34], Elan [3], Maude [6], and OBJ [11]. Rewriting is also a fundamental part of existing languages, including Tom [2],which integrates rewriting with Java.

2.2 Rewriting Logic

Rewriting logic [19, 18] is a computational logic built upon equational logic which provides support for concurrency. In equational logic, a number of *sorts* (types) and *equations* are defined. The equations specify which terms are considered to be equal. All equal terms can then be seen as members of the same equivalence class of terms, a concept similar to that from the λ calculus with equivalence classes based on α and β equivalence. Rewriting logic provides *rules* in addition to equations, used to transition between equivalence classes of terms. This allows for concurrency, where different orders of evaluation could lead to non-equivalent results, such as in the case of data races. The distinction between rules and equations is crucial for analysis, since terms which are equal according to equational deduction can all be collapsed into the same analysis state. Rewriting logic is connected to term rewriting in that the equations and rules of rewriting logic, of the form $l = r$ and $l \Rightarrow r$, respectively, can be transformed into term rewriting rules by orienting them properly (necessary because equations can be used for deduction in either direction), transforming both into $l \rightarrow r$. This provides a means of taking a definition in rewriting logic and a term and "executing" it.

2.3 Rewriting Logic Semantics

Rewriting logic semantics (RLS) [21, 22] builds upon the observation that programming languages can be defined as rewriting logic theories. By doing so, one gets essentially "for free" not only an interpreter and an initial model semantics for the defined language, but also a series of formal analysis tools obtained as instances of existing tools for rewriting logic. The work discussed in this paper has grown out of a style of RLS called *Continuation-Based Semantics* [21, 22] which allows the natural modeling of complex control flow constructs, like exceptions and continuations, by treating computations as first-class semantic entities.

2.4 K

K [30], a general notation and technique for defining computation, is based on insights developed in the rewriting logic semantics project [21, 22], with some concepts inspired by abstract state machines (ASMs) [12], the chemical abstract machine (CHAM) [10], and continuations [32]. [2] K provides some domain-specific abstractions and assumptions, exploited in this paper, to ease the definition of programming languages.

[2] The name K comes from the traditional name of the operator or cell containing the current control context, k.

The idea underlying language semantics in K is to represent the program configuration as a *computational structure*. This structure contains the context needed for the computation, with elements of the context represented as multisets or lists each stored inside a K *cell*. Contexts can also be hierarchical, with one cell containing others. The context generally includes standard items found in configurations in other formalisms, such as environments, stores, etc, as well as items specific to the given semantics, including such items as analysis results for a semantics focused on program analysis. One regularly used cell, referred to as k, represents the current computation as a $\curvearrowright$-separated list of computational tasks, such as $t_1 \curvearrowright t_2 \curvearrowright ... \curvearrowright t_n$. Another, $\top$, represents the entire computational structure. In the rest of the paper, the computational structure will be referred to as just a computation.

A K definition consists of two types of sentences: structural equations and rewrite rules. Structural equations carry no computational meaning; instead, borrowing a concept from CHAMs, structural equations can *heat* and *cool* computations. When a computation is heated, it breaks into smaller pieces, exposing subexpressions of more complex expressions for evaluation. Cooling reverses this process, reassembling the (potentially modified) pieces into a computation with the same "shape". The following are examples of structural equations, with heating represented as going from left to right and cooling from right to left:

$$a_1 + a_2 \rightleftharpoons a_1 \curvearrowright \square + a_2$$
$$\mathsf{if}\ b\ \mathsf{then}\ s_1\ \mathsf{else}\ s_2 \rightleftharpoons b \curvearrowright \mathsf{if}\ \square\ \mathsf{then}\ s_1\ \mathsf{else}\ s_2$$

Note that, unlike in evaluation contexts, $\square$ does not represent a context, but is instead part of the operator definition, providing visual intuition about what is being evaluated and where the result will go upon cooling; a different scheme could be used instead. The operators involving $\square$ above are $\square + \_$ (in the first equation) and $\mathsf{if}\ \square\ \mathsf{then\_else\_}$ (in the second).

Many structural equations can be automatically generated by annotating constructs in the language syntax with *strict* attributes: a *strict* construct generates the appropriate equations for each strict argument. If an operator is intended to be strict in only some of its arguments, then the positions of the strict arguments are listed as arguments of the *strict* attribute; for example, the two equations directly above correspond to the attributes *strict* for $\_ + \_$ (i.e., strict in all arguments, with the heating/cooling equations for the second operand not shown) and *strict*(1) for $\mathsf{if\_then\_else\_}$.

Rewrite rules represent actual steps of computation:

$$i_1 + i_2 \to i, \text{ where } i \text{ is the sum of } i_1 \text{ and } i_2$$
$$\mathsf{if\ true\ then}\ s_1\ \mathsf{else}\ s_2 \to s_1$$
$$\mathsf{if\ false\ then}\ s_1\ \mathsf{else}\ s_2 \to s_2$$
$$\langle\!|X\ =\ V|\!\rangle_k\ \langle\!\{(X,L)\}\!\rangle_{env}\ \langle\!\{(L,\_)\}\!\rangle_{mem} \to \langle\!|\cdot|\!\rangle_k\ \langle\!\{(X,L)\}\!\rangle_{env}\ \langle\!\{(L,V)\}\!\rangle_{mem}$$

Quite often structural equations would be used between applications of rewrite rules. For example, given an expression $a_1 + a_2$, the first equation for $\_ + \_$ can be applied left-to-right to "schedule" a_1 for processing (which may involve the

use of one or more rewrite rules); once evaluated to i_1, the equation is applied in reverse to "plug" the result back in context, leaving $i_1 + a_2$. a_2 would be handled similarly to a_1, yielding expression $i_1 + i_2$, which can then be processed using the first rewrite rule shown above. Note that this step, going from $i_1 + i_2$ to their sum, is not reversible, in contrast to the structural equations for + shown above. The last rule shows an example with multiple cells: here, the value V is being assigned to name X. The round bracket at the left, ⦇, represents the head of the list, forcing this rule to apply only when it will be the next step of this computation. The "pointed" bracket at the right, ⟫, represents the rest of the list, i.e. the remainder of the computation (intuitively, it is pointed as a reminder that the list keeps going in that direction). Multisets are bracketed with ⟪ and ⟫, indicating they conceptually "continue" in either direction. This is used here for both *env* and *mem*: *env* is a multiset of `Name` × `Location` pairs, while *mem* is a multiset of `Location` × `Value` pairs. Other notation includes _, which represents an unnamed value (like in many functional languages), and ·, representing the identity (here, the list identity). Given that, this rule states: when `X := V` is the next computational step in this computation, if `X` is at location `L` in the environment, change the value at location `L` in the store to `V` (while ignoring the current value), and then "dissolve" the current computation, leaving the next item in `k` (not shown, but to the "right" of ⟫) as the next computational step.

Context Transformers: To ensure that rules are modular, it should be possible to continue using a rule, unchanged, when parts of the context not mentioned in the rule are modified or replaced. Given a specific rule, the easiest case to deal with is when the subterm matched by a rule remains the same but the surrounding context changes (for instance, by adding a new top-level cell). This case is handled naturally by term rewrite systems, since it is possible to match a subterm of the term, leaving the rest unnamed in the rule. This handles many common cases, including the motivating example given in Section 1. However, this does not handle changes to the hierarchical organization of the context. For instance, adding threads to a language requires having multiple `k` cells, representing the computation occurring in each thread, but only one store, leading to a revised rule for assignment like the following:

$$\langle\!\langle ⦇X \;\; = \;\; V \rangle\!\rangle_k \; \langle\!\langle (X, L) \rangle\!\rangle_{env} \rangle\!\rangle_t \; \langle\!\langle (L, \_) \rangle\!\rangle_{mem} \rightarrow \langle\!\langle ⦇\cdot \rangle\!\rangle_k \; \langle\!\langle (X, L) \rangle\!\rangle_{env} \rangle\!\rangle_t \; \langle\!\langle (L, V) \rangle\!\rangle_{mem}$$

Beyond this, the configurations used in different languages will generally be quite different, and may be very complex. To allow rules to be reused, both as a language evolves and in other languages, K uses *context transformers*. Using context transformers, only those portions of the configuration actually used in a rule need to be mentioned. For instance, the assignment rule shown originally can remain as is, without the need to explicitly add the thread cell, used only to provide context for the match. To do this, the transformer uses the declared language configuration (shown in Section 3 in a `Language` module) to determine which cells, used only for context, have been elided; these cells can then be added automatically, using either variables or K brackets to represent unmentioned parts of the added cells. Several sanity conditions are used to ensure that a unique

transform is possible, including rules about valid paths between cells and the reuse of cell names. Planned K tool support will provide appropriate warnings in cases where ambiguity prevents the context transformers from deriving a unique transformed version of a rule.

3 The K Module System

While context transformers focus on making individual K rules modular, they do nothing to address the practical challenge of packaging up rules into reusable units. This is the purpose of the K module system.

The module system in K is being designed to support a general module syntax incorporating the entire range of functionality needed when defining the semantics of a language, including the definition of abstract syntax, configuration items, the semantics of language features, and the final collection of features that make up a specific language. In theory, this would allow a single, monolithic module to include definitions of all aspects of the semantics. However, to provide for a better separation of these constructs into more granular modules, and to allow for construct-specific defaults and syntax, specialized module formats for various constructs are being defined, with a translation into the more general syntax. These module formats are illustrated with fragments of the definition of a simple imperative language, IMP. A complete definition of IMP, without the extensions presented at the end of this section, is given in Appendices A and B; note that the definition given in this section differs slightly to better illustrate features of the module system.

3.1 Semantic Entities

```
module Int
  imports K/Value, K/Int .
  subsort Int < Value .
end module
```

Fig. 1. Integer Values

Semantic entities in K definitions include configuration items, such as environments and stores, and sorts or operations used during computations, such as computation items and values. A simple example is shown in Figure 1, which uses subsorting to allow K integers (modules starting with `K/` are built-ins) to be treated as K values. By default, this declaration is available in any other module that imports `Int`.

```
module Env
  imports K/Name .
  requires sort Loc .
  sortalias Env = Map(Name,Loc) .
  var Env[0-9']* : Env .
end module
```

Fig. 2. Environments

Another example is shown in Figure 2. This shows the definition of an environment, which provides a mapping from names to locations (a store then maps locations to values; the separation easily allows features like nested scopes and reference parameters for functions). Like in Figure 1, an existing K definition, in this case for sort `Name`, is imported. Instead of similarly importing a specific definition of locations (sort `Loc`), module `Env` uses `requires`, meaning that, when the language is finally assembled, one module must provide sort `Loc`. This allows the module to state a requirement without stating the module that satisfies that requirement, allowing different modules to be used in different languages. Since

K provides lists, multisets, and maps by default, we can immediately refer to maps from sort `Name` to sort `Loc`; `sortalias` lets us give this sort a name, `Env`, which can then be used in the remainder of the definition. The `var` declaration allows the definition of a variable pattern: `Env`, followed by 0 or more numbers or primes, will be used to represent entities of sort `Env` (e.g., `Env`, `Env8`, `Env'`, etc.). Variables used in modules that import `Env` and that match this pattern will then be identified as being of sort `Env`.

3.2 Abstract Syntax

```
module Exp/AExp[Syntax]
  imports Exp[Syntax] with
      { sort Exp renamed AExp } .
  # sort AExp . subsort AExp < Exp .
  var AE[0-9'a-zA-Z]* : AExp .
end module
```

Fig. 3. Arithmetic Expressions

Before defining the semantics of language constructs, the abstract syntax of those constructs needs to be defined. This is done using abstract syntax modules, which are defined using a tag of `[Syntax]` after the module name. A first example of an abstract syntax module is the syntax for arithmetic expressions, shown in Figure 3. One way to define the sort of arithmetic expressions would be to define a new sort which could be made a subsort of `Exp`, illustrated in a comment in the module (comments start with `#`); here, instead, the sort `Exp`, imported from module `Exp`, is renamed to `AExp` using a sort renaming directive on the import of module `Exp`. A var pattern to refer to arithmetic expressions is then defined.

```
module Exp/AExp/Plus[Syntax]
  imports Exp/AExp[Syntax] .
  _+_ : AExp AExp -> AExp .
end module
```

Fig. 4. Plus Expressions

A second abstract syntax module, defining the addition construct, is shown in Figure 4. Syntax is defined using mixfix notation with an algebraic notation similar to that used in Maude or SDF (although note that `op` is not required on syntax definitions). To increase modularity, it is recommended that each module define only one language construct, although it is possible to define multiple constructs in the same module.

3.3 Semantic Rules

Once the syntax has been defined, the semantics of each construct need to be defined as well. One explicit goal of the module system is to allow different semantics to be easily defined for each language construct. For instance, it should be possible to define a standard dynamic/execution semantics, a static/typing semantics, and potentially other semantics manipulating different notions of value (for instance, various notions of abstract value used during analysis).

```
module Exp/AExp/Plus[Dynamic]
  imports Exp/AExp/Plus[Syntax]
    with { op _+_ now strict,
                 extends + [Int * Int -> Int] } .
end module
```

Fig. 5. Dynamic Semantics: Plus

Figure 5 shows an example of a module defining the dynamic semantics of a language feature, here integer addition. Normally a semantics module will implicitly import the related syntax module. Here, since we are modifying the attributes on an imported operator, we need to explicitly import the syntax module. Two attributes are modified. First, we

note that the operator is now strict in all arguments, which will automatically generate the structural heating and cooling equations. Second, we use extends to automatically "hook" the semantics of the feature to the builtin definition of integer addition. This completely defines integer addition in the language, so no rules are needed.

```
module Exp/AExp/Plus[Static] is
  imports Exp/AExp/Plus[Syntax] with { op _+_ now strict } .
  imports Types .
  rl int + int => int .
  rl T + T' => fail [where T =/= int or T' =/= int] .
end module
```

Fig. 6. Static Semantics: Plus

Figure 6 shows semantics for the same feature, but this time the static semantics (for type checking) are defined. Like in Figure 5, the operator for plus is changed to be strict. In this case, though, the values being manipulated are types, not integers, so we also need to import the types and use them in the two rules shown. Here, the first rule is for when an expression is type correct: the two operands are both integers, so the result of adding them is also an integer. If one of the operands is not an integer (checked in the side-condition), the rule will cause a type called fail, representing a type error, to propagate.[3]

```
module Stmt/Block[Dynamic] is
  imports Stmt[Syntax], K/K, Env .
  rl k(| begin S end          |> env(| Env |)
  => k(| S -> restoreEnv(Env) |> env(| Env |) .
end module
```

Fig. 7. Dynamic Semantics: Blocks

Finally, Figure 7 shows the dynamic semantics of blocks. Here, no changes are made to the imported syntax, so there is no need to import the `Stmt/Block[Syntax]` module explicitly. In this language, blocks provide for nested scoping, so we want to ensure that the current environment is restored after the code inside the block executes. This is done by capturing the current environment, `Env`, and placing it on the computation in a `restoreEnv` computation item. The rule for `restoreEnv`, not shown here, will replace the current environment with its saved environment when it becomes the first item in the computation.

3.4 Language Definitions

Once the semantic entities, abstract syntax, and language semantics have been defined, they can be assembled into a language module, tagged `Language`. An example is shown in Figure 8. The line `config =` defines the language configuration as a multiset, with each K cell given a name (such as `store` or `env`) and the sort of information in the cell (such as `Store`

```
module Imp[Language]
  imports K/Configuration, K/K, K/Location,
          K/Value, Env, Store, Int, Bool .
  config = top(store(Store) env(Env)
           k(K) nextLoc(Loc)) .

  op [[_]] : K -> Configuration .
  eq [[ K ]] = top(store(empty) env(empty)
               k(K) nextLoc(initLoc)) .

  imports type=Syntax Exp/AExp/Num, Exp/BExp/Bool .
  imports type=Dynamic Exp/AExp/Name, Exp/AExp/Plus,
    Exp/BExp/LessThanEq, Exp/BExp/Not, Exp/BExp/And,
    Stmt/Sequence, Stmt/Assign, Stmt/IfThenElse,
    Stmt/While, Stmt/Halt, Pgm .
end module
```

Fig. 8. Language Definition: IMP

[3] An alternative would be to issue an error message and return the expected type in the hope of finding additional errors

or `Env`). Cells can be nested, to represent the hierarchies of information that can be formed. Next, the `[[_]]` operator initializes this configuration, given an initial computation (`K`) representing the program to run. Finally, all the modules used in the semantics are imported. `type=Dynamic` is a directive that states that all modules in this `imports` are tagged with the `Dynamic` tag, and is equivalent to `imports Exp/AExp/Name[Dynamic], Exp/AExp/Plus[Dynamic]`, etc.

3.5 Taking Advantage of Modularity

The goal of ensuring that K is modular is to allow defined language features to be reused in new languages and in extensions to existing languages. To illustrate this, two extensions to IMP, one for exceptions and one for procedures, are defined, generating two new versions of IMP. These extensions are then combined to create a third version of IMP, showing that the existing definitions can be directly leveraged. To save space, the following are not shown: abstract syntax modules for the new features; the `Language` modules for IMP with just exceptions or with just procedures; and imports clauses in module definitions.

Exceptions: The exceptions extension assumes an abstract syntax similar to that for Java: a `try/catch` statement is used to specify an exception handler, while `throw` is used to manually throw an exception. Figure 9 shows the semantics needed for exceptions. In the case of a `try/catch`, the information needed for the handler is saved with `addHandler` before the `try` statement is executed. If the `try` body (`S`) is executed without throwing an exception, `removeHandler` removes the handler information. If an exception is thrown, either implicitly or with `throw`, the handler will be triggered, in the case of `throw` through using `handleException`.

```
module Stmt/TryCatch[Dynamic]
  requires op addHandler : Name Stmt -> Computation .
  requires op removeHandler : -> Computation  .
  rl k(| try S catch X in S' |>
  => k(| addHandler(X,S') -> S -> removeHandler |> .
end module
module Stmt/Throw[Dynamic]
  requires op handleException : -> Computation .
  rl k(| throw V |> => k(| V -> handleException |> .
end module
```

Fig. 9. Exception Semantics

Figure 10 then shows the state operations used in the exception semantics in Figure 9. An exception handler is defined as a triple, with a computation, an environment, and arbitrary other state (K cells). These are stored in an exception handler stack, defined as a list of exception handlers. Rules then provide semantics for the operations: `removeHandler` just pops the stack, while `addHandler` creates the exception handler (`assignTo` will take the thrown value, which will be at the head of the

```
module Exceptions/State
  sortalias ExHandler = Tuple(K,Env,State) .
  sortalias ExStack = List(ExHandler) .
  rl k(| removeHandler |> es(| EH |>
  => k(| . |> es(| . |> .
  rl k(| addHandler(X,S) |> env(| Env |)
       cn(| es(| ES |) CN |)
  => k(| . |> env(| Env |)
       cn(| es(| [assignTo(X) -> S -> restoreEnv(Env),
                  Env, CN] ES |) CN |) .
  rl k(| V -> handleException |>
       env(| _ |) cn(| es(| [K,Env,CN] |> |>
  => k(| V -> K |) env(| Env |) cn(| es(| . |> CN |) .
end module
```

Fig. 10. Exception State Manipulation

computation, and assign it to the identifier given in the `catch` statement, while `restoreEnv` will then restore the environment back to the given environment, removing this name mapping). Note the use of two new K cells in these two rules: `es`, for the exception stack, and `cn`, for control context information, like that used in exception handlers (this extra level of grouping will prove useful later). Finally, `handleException` will use the handler to handle the exception, restoring the saved computation, environment, and control context in the process.

Procedures: The semantics for procedures are similar to those for exceptions. When a procedure is called with `call`, it will use `invoke` to invoke the procedure and save the current state. The semantics for `return` are similar to `throw`, using `popCallStack` to remove the current procedure context and restore state saved at the time of the call. Figure 11 shows the semantics for both `call` and `return`.

```
module Stmt/Call[Dynamic]
  requires op invoke : Name ValueList -> K .
  rl k(| call X(VL) |>
  => k(| invoke(X,VL) |> .
end module
module Stmt/Return[Dynamic]
  requires op popCallStack : -> K .
  rl k(| return |> => k(| popCallStack |> .
end module
```

Fig. 11. Procedure Semantics

Figure 12 then shows the state manipulation rules, similar in many ways to those for exceptions. Note that there are two new K cells here as well: `cs`, for the call stack, and `pm`, for the procedure map (from procedure names to procedure definitions). `cn`, for control context, is also used. `invoke` saves the current computation, environment, and other control context in the call stack while assigning argument values (`VL`) to parameter names (`XL`) and then running the procedure body (`S`), all in the context of an empty environment (i.e., there are no global variables). The procedure definition is looked up as part of a side condition (with `where` in the brackets following the rest of the rule). `popCallStack` restores the saved control context, environment, and computation, representing the return from the procedure.

```
module Procedures/State
  sortalias CallState = Tuple(K,Env,State) .
  sortalias CallStack = List(CallState) .
  rl k(| invoke(X,VL) -> K |) env(| Env |)
       cn(| cs(| CS |) CN |) pm(PM)
  => k(| VL -> assignTo(XL) -> S |) env(| . |)
       cn(| cs(| [K,Env,CN] CS |) CN |) pm(PM)
     [where proc(XL,S) := lookup(PM,X)] .
  rl k(| popCallStack |> env(| _ |)
       cn(| cs(| [K,Env,CN] |> |>
  => k(| K |) env(| Env |) cn(| cs(| . |> CN |) .
end module
```

Fig. 12. Procedure State Manipulation

Combining Exceptions and Procedures: It should be possible to reuse the definitions of exceptions and procedures without needing to revisit each. This can in fact be done, without requiring any changes to the defined language features. Figure 13 shows the language module, including K cells, that extends IMP with both procedures and exceptions. The use of cell `cn` to group context information, along with the use of context transformers to transform rules that mention both `k` and (for instance) `cs` into ones that also use `cn` and the other context held therein allows the language to be assembled without modifying any existing module. Note that this is not always possible, as different language features may need to be aware of one another. For instance, a definition of a loop `break` feature

```
module ImpWCallWExceptions[Language]
  imports K/Configuration, K/K, K/Location, K/Value,
          Env, Store, Int, Bool, Procedures/State,
          Exceptions/State .

  config = top(store(Store) env(Env) k(K) nextLoc(Loc) cn(cs(CallStack)
             es(ExStack)) pm(ProcMap)) .

  op [[_,_]] : ProcList K -> Configuration .
  eq [[PL,K]] = top(store(empty) env(empty) k(processEach(PL) -> K)
                  nextLoc(initLoc) cn(cs(empty) es(empty)) pm(empty)) .

  imports type=Syntax Exp/AExp/Num, Exp/BExp/Bool .
  imports type=Dynamic Exp/AExp/Name, Exp/AExp/Plus,
    Exp/BExp/LessThanEq, Exp/BExp/Not, Exp/BExp/And,
    Stmt/Sequence, Stmt/Assign, Stmt/IfThenElse,
    Stmt/While, Stmt/Halt, Stmt/Call, Stmt/Return, Stmt/TryCatch,
    Stmt/Throw, Procedure, Pgm.
end module
```

Fig. 13. IMP with Procedures and Exceptions

may need to be aware of functions, since it is generally not possible to use `break` inside a called function to break out of a loop inside the callee.

4 Related Work

Modularity has long been a topic of interest in the language semantics community. Listed below are some of the more significant efforts, including comparisons with the work described in this paper where appropriate.

Action Semantics: One focus of Action Semantics [24] has been on creating modular definitions. The notation for writing action semantics definitions uses a module structure, while language features use *facets* to separate different language construct "concerns", such as updating the store or communicating between processes. A number of tools have been created for working with modular Action Semantics definitions, such as ASD [35], the Action Environment [33], the Maude Action Tool [7], an implementation using Montages [1], and Modular Monadic Action Semantics [37]. Other work has focused specifically on ensuring modules can be easily reused without change, both by using small, focused modules [8] (the approach taken in the K module system) and by creating a number of simpler reusable constructs generic to a large number of languages [27, 15].

ASMs: Montages [16] provides a modular way to define language constructs using Abstract State Machines (ASMs) [14, 31]. Each Montage (i.e., module) combines a graphical depiction of a language construct with information on the static and dynamic semantics of the feature. This has the advantage of keeping all information on a feature in one place, but limits extensibility, since it is not possible (as it is in K) to provide multiple types of dynamic or static semantics to the same feature without creating a new Montage.

Denotational Semantics: One effort to improve modularity in denotational semantics definitions has been the use of Monads [23]. This has been most evident in work on modular, semantics-based definitions of language interpreters and compilers, especially in the context of languages such as Haskell [36, 17] and Scheme [9]. Monads have also been used to improve the modularity of other semantic formalisms, such as Modular Monadic Action Semantics [37], which provided a monadic semantics in place of the original, non-modular SOS semantics underlying prior versions of Action Semantics [24].

MSOS: The focus of MSOS [25, 26] has been on keeping the benefits of SOS definitions while defining rules in a modular fashion. This is done by moving information stored in SOS configurations, such as stores, into the labels on transition rules, which traditionally have not been used in SOS definitions of languages. This, along with techniques that allow parts of the label to be elided if not used by a rule, allow the same rule to be used both when unrelated parts of the configuration change and when the rule is introduced into a language with a different configuration. A recent innovation, Implicitly-Modular SOS (I-MSOS) [28], allows more familiar SOS notation while still providing the benefits of MSOS.

Rewriting Logic Semantics: Beyond the work done on K, Maude has also been used as a platform to experiment with other styles of semantics, enabling the creation of modular language definitions. This includes work on action semantics, with the Maude Action Tool cited above, and MSOS, using the Maude MSOS Tool [5]. Work on defining Eden [13], a parallel variant of Haskell, has focused on modularity to allow for experimentation with the degree of parallelism and the scheduling algorithm used to select processes for execution. General work on modularity of rewriting logic semantics definitions [20, 4] has focused on defining modular features that need not change as a language is extended.

5 Conclusion and Future Work

In this paper we have presented ongoing work on modularity in K language definitions. This includes work on the modularity of individual K rules, ensuring they can be defined once and reused in a variety of contexts, and work on an actual module system for K, providing a technique to easily package and reuse individual language features while building a language.

One major, ongoing component of this work is developing tool support for the module system. Although small modules improve reuse, the large number of modules this leads to can make it challenging to work with language definitions, something noted in similar work on tool support for Action Semantics [33]. For K, work on tool support includes the ongoing development of an Eclipse plugin to provide a graphical environment for the creation and manipulation of K modules. This will initially include editor support, a graph view of module dependencies, and the ability to view both the language features used to define a language and the various semantics defined for a specific language feature. Longer-term goals

include the graphical assembly of language configurations and links to an online database of reusable modules. Another part of this work is moving over existing language definitions to the new, modular format. This has already started for a number of pedagogical languages defined in K that are used in the classroom; work on larger languages is waiting on improved tool support.

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A The K Definition of IMP

Figure 14 shows the K definition of the IMP language, a version of which has been used as the running example for the presentation of the module system in this paper. This definition is discussed more fully in a technical report on K [30].

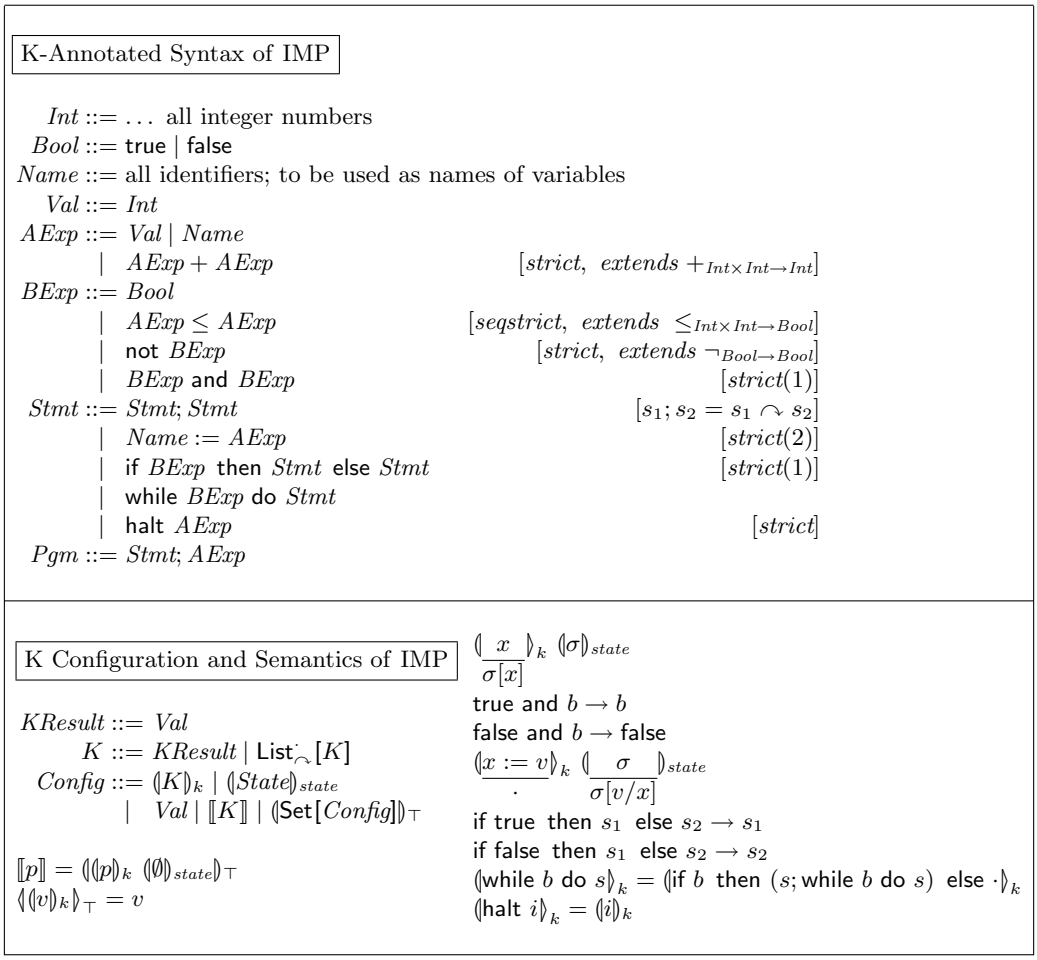

Fig. 14. K definition of IMP.

B The Modular K Definition of IMP

A number of modules make up the definition of the IMP language. The first modules shown below make up semantic entities used in the definition.

```
module Env
  requires sort Name, sort Loc .
  sortalias Env = Map(Name,Loc) .
  var Env[0-9']* : Env .
end module

module Store
  requires sort Loc, sort Value .
  sortalias Store = Map(Loc,Value) .
  var Store[0-9']* : Store .
end module
```

```
module Int
  imports K/Int .
  requires sort Value .
  subsort Int < Value .
end module

module Bool
  sort Bool .
  ops true false : -> Bool .
end module
```

The next modules define the abstract syntax for IMP. This includes sorts for arithmetic expressions, boolean expressions, statements, and programs, as well as a number of syntactic entities (i.e., productions). Note that modules can import other modules of the same "type" (`Syntax`, `Dynamic`, etc) without needing to specify the type. If this would lead to an ambiguous import a warning message will be generated.

```
module Exp/AExp[Syntax]
  sort AExp .
  var AE[0-9'a-zA-Z]* : AExp .
end module

module Exp/AExp/Num[Syntax]
  imports Int, Exp/AExp .
  subsort Int < AExp .
end module

module Exp/AExp/Name[Syntax]
  imports Exp/AExp .
  requires sort Name .
  subsort Name < AExp .
end module

module Exp/AExp/Plus[Syntax]
  imports Exp/AExp .
  _+_ : AExp AExp -> AExp .
end module

module Exp/BExp[Syntax]
  sort BExp .
  var BE[0-9'a-zA-Z]* : BExp .
end module

module Exp/BExp/Bool[Syntax]
  imports Bool, Exp/BExp .
  subsort Bool < BExp .
end module

module Exp/BExp/LessThanEq[Syntax]
  imports Exp/AExp, Exp/BExp .
  _<=_ : AExp AExp -> BExp .
end module

module Exp/BExp/Not[Syntax]
  imports Exp/BExp .
  not_ : BExp -> BExp .
end module
```

```
module Exp/BExp/And[Syntax]
  imports Exp/BExp .
  _and_ : BExp BExp -> BExp .
end module

module Stmt[Syntax]
  sort Stmt .
end module

module Stmt/Sequence[Syntax]
  imports Stmt .
  _;_ : Stmt Stmt -> Stmt .
end module

module Stmt/Assign[Syntax]
  imports Stmt, Exp/AExp .
  requires sort Name .
  _:=_ : Name AExp -> Stmt .
end module

module Stmt/IfThenElse[Syntax]
  imports Stmt, Exp/BExp .
  if_then_else_ : BExp Stmt Stmt -> Stmt .
end module

module Stmt/While[Syntax]
  imports Stmt, Exp/BExp .
  while_do_ : BExp Stmt -> Stmt .
end module

module Stmt/Halt[Syntax]
  imports Stmt, Exp/AExp .
  halt_ : AExp -> Stmt .
end module

module Pgm[Syntax]
  imports Stmt, Exp/AExp .
  sort Pgm .
  _;_ : Stmt AExp -> Pgm .
end module
```

Using the abstract syntax, a number of modules are used to define the evaluation semantics, with one semantics module per language feature. As a

reminder, a semantics module will automatically import the syntax module of the same name; explicit imports of syntax modules are used in cases where attributes, such as strictness, need to be changed. Added strictness information will cause heating and cooling rules to be automatically generated. The `seqstrict` attribute, used on less than, is identical to `strict`, except it enforces a left to right evaluation order on the arguments.

```
module Exp/AExp/Name[Dynamic]
  requires sort Name, sort Loc, sort Value .
  imports Env, Store, K/K .
  rl k(| X |> env<| (X,L) |> store<| (L,V) |>
  => k(| V |> env<| (X,L) |> store<| (L,V) |> .
end module

module Exp/AExp/Plus[Dynamic]
  imports Exp/AExp/Plus[Syntax]
     with { op _+_ now strict,
              extends + [Int * Int -> Int] } .
end module

module Exp/BExp/LessThanEq[Dynamic]
  imports Exp/BExp/LessThanEq[Syntax]
    with { op _<=_ now seqstrict,
             extends <= [Int * Int -> Bool] } .
end module

module Exp/BExp/Not[Dynamic]
  imports Exp/BExp/Not[Syntax]
    with { op not_ now strict,
             extends not [Bool -> Bool] } .
end module

module Exp/BExp/And[Dynamic]
  imports Exp/BExp/And[Syntax]
    with { op _and_ now strict(1) } .
  rl true and BE => BE .
  rl false and BE => false .
end module

module Stmt/Sequence[Dynamic]
  eq S ; S' = S -> S' .
end module
```

```
module Stmt/Assign[Dynamic]
  imports Stmt/Assign[Syntax]
    with { op _:=_ now strict(2) } .
  requires sort Name, sort Value, sort Loc .
  imports K/K, Env, Store .
  rl k(| X := V |> env<| (X,L) |>
     store<| (L,_) |>
  => k(| .      |> env<| (X,L) |>
     store<| (L,V) |> .
end module

module Stmt/IfThenElse[Dynamic]
  imports Stmt/IfThenElse[Syntax]
    with { op if_then_else_ now strict(1) } .
  imports Bool .
  rl if true then S else S' => S .
  rl if false then S else S' => S' .
end module

module Stmt/While[Dynamic]
  imports Stmt/IfThenElse, Exp/BExp[Syntax] .
  eq k(| while BE do S |>
   = k(| if BE then (S ;
         while BE do S) else . |> .
end module

module Stmt/Halt[Dynamic]
  imports Stmt/Halt[Syntax]
    with {op halt_ now strict } .
  imports Int .
  eq k(| halt i |> = k(| i |) .
end module

module Pgm[Dynamic]
  requires sort Value .
  eq top (| k(| V |) |> = V .
end module
```

Finally, the various modules are assembled together into a language module, representing the entire programming language.

```
module Imp[Language]
  imports K/Configuration, K/K, K/Location, K/Value,
          Env, Store, Int, Bool .
  config = top(store(Store) env(Env) k(K) nextLoc(Loc)) .

  op [[_]] : K -> Configuration .
  eq [[ K ]] = top(store(empty) env(empty) k(K) nextLoc(initLoc)) .

  imports type=Syntax Exp/AExp/Num, Exp/BExp/Bool .
  imports type=Dynamic Exp/AExp/Name, Exp/AExp/Plus, Exp/BExp/LessThanEq, Exp/BExp/Not,
    Exp/BExp/And, Stmt/Sequence, Stmt/Assign, Stmt/IfThenElse, Stmt/While, Stmt/Halt, Pgm .
end module
```