Suppose we deposit \$1000 in a savings account that pays 5% interest compounded annually for 142 years—how much will we have in total savings?

- Why NOT: $\cancel{1000 + 1000 \times .05 \times 142 = 8100}$?

Suppose we deposit \$1000 in a savings account that pays 5% interest compounded annually for 142 years—how much will we have in total savings?

- Why NOT: $\cancel{1000 + 1000 \times .05 \times 142 = 8100}$?
- year 1: $1000 + 1000 \times .05 = 1050$
- year 2: year 1 total + year 1 total $\times .05$
  could do this 142 times but would take forever—apply algebra!

Suppose we deposit \$1000 in a savings account that pays 5% interest compounded annually for 142 years—how much will we have in total savings?

- Why NOT: $\cancel{1000 + 1000 \times .05 \times 142 = 8100}$?
- year 1: $1000 + 1000 \times .05 = 1050$
- year 2: year 1 total + year 1 total $\times .05$
  could do this 142 times but would take forever—apply algebra!
- rewrite year 1 and factor out the common term of 1000:
  $1000 \times 1 + 1000 \times .05 =$

Suppose we deposit \$1000 in a savings account that pays 5% interest compounded annually for 142 years—how much will we have in total savings?

- Why NOT: $\cancel{1000 + 1000 \times .05 \times 142 = 8100}$?
- year 1: $1000 + 1000 \times .05 = 1050$
- year 2: year 1 total + year 1 total $\times .05$
  could do this 142 times but would take forever—apply algebra!
- rewrite year 1 and factor out the common term of 1000:
  $1000 \times 1 + 1000 \times .05 = 1000(1 + .05)$

Suppose we deposit \$1000 in a savings account that pays 5% interest compounded annually for 142 years—how much will we have in total savings?

- Why NOT: $\cancel{1000 + 1000 \times .05 \times 142 = 8100}$?
- year 1: $1000 + 1000 \times .05 = 1050$
- year 2: year 1 total + year 1 total $\times .05$
  could do this 142 times but would take forever—apply algebra!
- rewrite year 1 and factor out the common term of 1000:
  $1000 \times 1 + 1000 \times .05 = 1000(1 + .05)$
- year 2: year 1 total + year 1 total $\times .05$
  $= 1000(1 + .05) \times 1 + 1000(1 + .05) \times .05$

Suppose we deposit \$1000 in a savings account that pays 5% interest compounded annually for 142 years—how much will we have in total savings?

- Why NOT: $\cancel{1000 + 1000 \times .05 \times 142 = 8100}$?
- year 1: $1000 + 1000 \times .05 = 1050$
- year 2: year 1 total + year 1 total $\times .05$
  could do this 142 times but would take forever—apply algebra!
- rewrite year 1 and factor out the common term of 1000:
  $1000 \times 1 + 1000 \times .05 = 1000(1 + .05)$
- year 2: year 1 total + year 1 total $\times .05$
  $= 1000(1 + .05) \times 1 + 1000(1 + .05) \times .05$
  factor out the common term of $1000\ (1 + .05)$
  $= 1000\ (1 + .05)\ (1 + .05)$

Suppose we deposit \$1000 in a savings account that pays 5% interest compounded annually for 142 years—how much will we have in total savings?

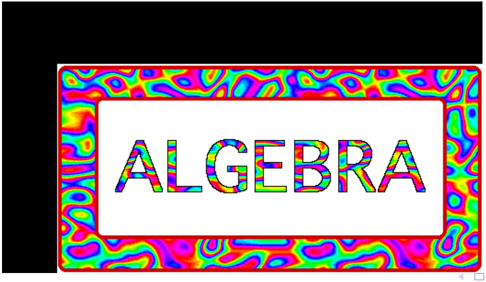
- Why NOT: $\cancel{1000 + 1000 \times .05 \times 142 = 8100}$?
- year 1: $1000 + 1000 \times .05 = 1050$
- year 2: year 1 total + year 1 total $\times .05$
  could do this 142 times but would take forever—apply algebra!
- rewrite year 1 and factor out the common term of 1000:
  $1000 \times 1 + 1000 \times .05 = 1000(1 + .05)$
- year 2: year 1 total + year 1 total $\times .05$
  $= 1000(1 + .05) \times 1 + 1000(1 + .05) \times .05$
  factor out the common term of 1000 (1 + .05)
  =1000 (1 + .05) (1 + .05)
  use exponent rule to simplify
  $= 1000(1 + .05)^2$

Suppose we deposit \$1000 in a savings account that pays 5% interest compounded annually for 142 years—how much will we have in total savings?

- Why NOT: $\cancel{1000 + 1000 \times .05 \times 142 = 8100}$?
- year 1: $1000 + 1000 \times .05 = 1050$
- year 2: year 1 total + year 1 total $\times$.05
  could do this 142 times but would take forever—apply algebra!
- rewrite year 1 and factor out the common term of 1000:
  $1000 \times 1 + 1000 \times .05 = 1000(1 + .05)$
- year 2: year 1 total + year 1 total $\times$.05
  $= 1000(1 + .05) \times 1 + 1000(1 + .05) \times .05$
  factor out the common term of 1000 (1 + .05)
  =1000 (1 + .05) (1 + .05)
  use exponent rule to simplify
  $= 1000(1 + .05)^2$
- year 142?

How We Derived the Lump Sum Formula

We obtained the general formula for lump sum using the total from the year before to calculate the principal and interest for the next year. This process works fine, but is too difficult to use when the number of years is large. So we looked for a way to obtain a simplified formula. We looked for the commonality and recognized the repeated appearance of (1+rate) after factoring. Once we found this pattern, we used it to find a simplified formula.



Suppose we deposit \$1000 in a savings account that pays 5% interest compounded annually for 142 years—how much will we have in total savings? $1000(1+.05)^{142}$

Now, suppose we deposit \$1000 in a savings account that pays 5% interest compounded monthly for 142 years—how much will we have in total savings?

5% interest compounded monthly means 5% is the annual rate so the monthly rate, the periodic rate, is $\frac{.05}{12}$

a) $1000(1+\frac{.05}{12})^{142}$
b) $1000(1+\frac{.05}{12})^{1704}$
c) other

Suppose we deposit \$1000 in a savings account that pays 5% interest compounded annually for 142 years—how much will we have in total savings? $1000(1+.05)^{142}$

Now, suppose we deposit \$1000 in a savings account that pays 5% interest compounded monthly for 142 years—how much will we have in total savings?

5% interest compounded monthly means 5% is the annual rate so the monthly rate, the periodic rate, is $\frac{.05}{12}$

a) $1000(1+\frac{.05}{12})^{142}$
b) $1000(1+\frac{.05}{12})^{1704}$
c) other

Which is better interest in this scenario, compounding annually, compounding monthly, or are they the same?

Suppose we deposit \$1000 in a savings account that pays 5% interest compounded annually for 142 years—how much will we have in total savings? $1000(1+.05)^{142}$

Now, suppose we deposit \$1000 in a savings account that pays 5% interest compounded monthly for 142 years—how much will we have in total savings?

5% interest compounded monthly means 5% is the annual rate so the monthly rate, the periodic rate, is $\frac{.05}{12}$

a) $1000(1+\frac{.05}{12})^{142}$
b) $1000(1+\frac{.05}{12})^{1704}$
c) other

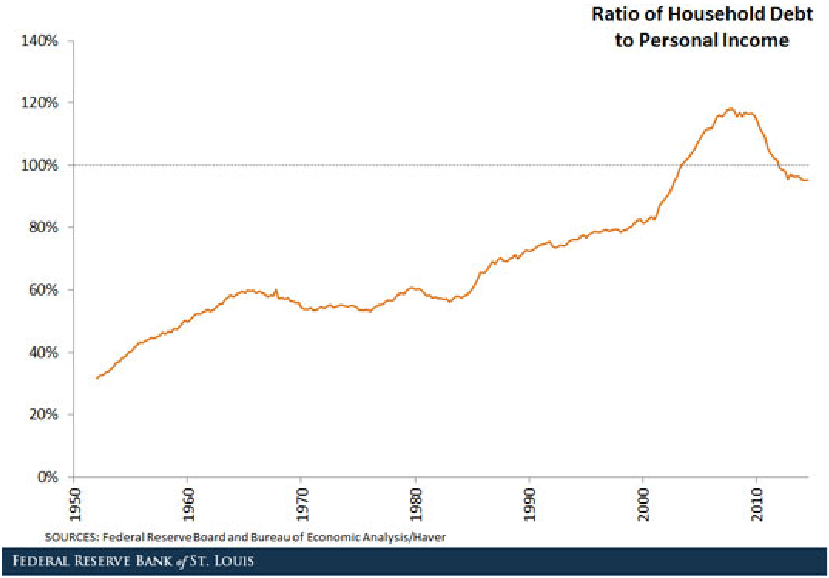
Which is better interest in this scenario, compounding annually, compounding monthly, or are they the same?

total = lump(1 + periodic rate)$^{\#\text{times we actually compound}}$

interest = total $-$ amount put in as a lump sum

The role of chance and probability in financial forecasts

- all financial forecasts, whether about the specifics of a local business, like sales growth, or predictions about the global economy as a whole, are informed guesses based on historical data and other analyses. Historical data is all we have to go on and there is no guarantee that the conditions in the past will persist into the future.
- debt-to-income ratio over time



www.stlouisfed.org/on-the-economy/2015/march/mortgage-debt-and-the-great-recession

Real-life situation: Past student was told that her certificate of deposit (CD) will be compounded monthly at 8% for 8 months, and is told that this 8% will apply each and every month (i.e. is the monthly rate). Let's say that she put in \$1000. How much would her CD be worth at the end of 8 months if the annual rate was indeed $8 \times 12 = 96\%$ instead of 8% that every other bank would mean?

a) $1000(1 + .08)^8$
b) $1000(1 + \frac{.08}{8})^8$
c) $1000(1 + \frac{.08}{12})^{8\times12}$
d) $1000(1 + \frac{.08}{12})^8$
e) none of the above

Real-life situation: Past student was told that her certificate of deposit (CD) will be compounded monthly at 8% for 8 months, and is told that this 8% will apply each and every month (i.e. is the monthly rate). Let's say that she put in \$1000. How much would her CD be worth at the end of 8 months if the annual rate was indeed $8 \times 12 = 96\%$ instead of 8% that every other bank would mean?

a) $1000(1+.08)^8$
b) $1000(1+\frac{.08}{8})^8$
c) $1000(1+\frac{.08}{12})^{8\times 12}$
d) $1000(1+\frac{.08}{12})^8$
e) none of the above

What did the bank really mean?

Real-life situation: Past student was told that her certificate of deposit (CD) will be compounded monthly at 8% for 8 months, and is told that this 8% will apply each and every month (i.e. is the monthly rate). Let's say that she put in \$1000. How much would her CD be worth at the end of 8 months if the annual rate was indeed $8 \times 12 = 96\%$ instead of 8% that every other bank would mean?

a) $1000(1 + .08)^8$
b) $1000(1 + \frac{.08}{8})^8$
c) $1000(1 + \frac{.08}{12})^{8\times 12}$
d) $1000(1 + \frac{.08}{12})^8$
e) none of the above

What did the bank really mean?
total savings plus interest = $1000(1 + \frac{.08}{12})^8$
interest = $1000(1 + \frac{.08}{12})^8 - 1000$

interest = total $-$ amount put in as a lump sum

total = lump (1 + periodic rate) $^{\#\text{times we actually compound}}$

lump amount, time length, rate, or number of times compounding per year might be the unknown

interest = total $-$ amount put in as a lump sum

total = lump (1 + periodic rate) $^{\#\text{times we actually compound}}$

lump amount, time length, rate, or number of times compounding per year might be the unknown

- Intro to Goal Seek in Excel spreadsheet via seeing how long it will take to double our money using her rate. $2000 = 1000(1 + \frac{.96}{12})^{?}$
- cell is denoted by its column, row: A1
- formulas in Excel start with =

interest = total $-$ amount put in as a lump sum

total = lump (1 + periodic rate) $^{\#\text{times we actually compound}}$

lump amount, time length, rate, or number of times compounding per year might be the unknown

- Intro to Goal Seek in Excel spreadsheet via seeing how long it will take to double our money using her rate. $2000 = 1000(1 + \frac{.96}{12})^{?}$
- cell is denoted by its column, row: A1
- formulas in Excel start with =

	A	B	C
1	total savings	months	years
2	=1000*(1+.96/12)^B2		=B2/12

What-If Analysis... Goal Seek:

Set cell: A2

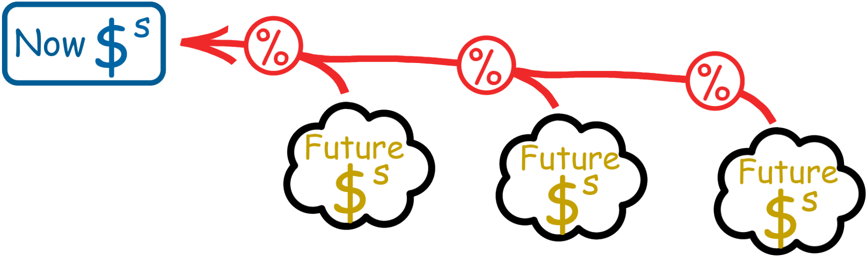
To value: 2000

By changing cell: B2

How much should we put in now as a lump sum if we want the future value (FV) to be \$500 after 14 years of an account paying 1% compounded monthly (i.e. what is the present value (PV) of the account)?

How much should we put in now as a lump sum if we want the future value (FV) to be \$500 after 14 years of an account paying 1% compounded monthly (i.e. what is the present value (PV) of the account)?

$$500 = PV(1+\tfrac{.04}{12})^{14\cdot 12} = PV(1+\tfrac{.04}{12})^{168}$$

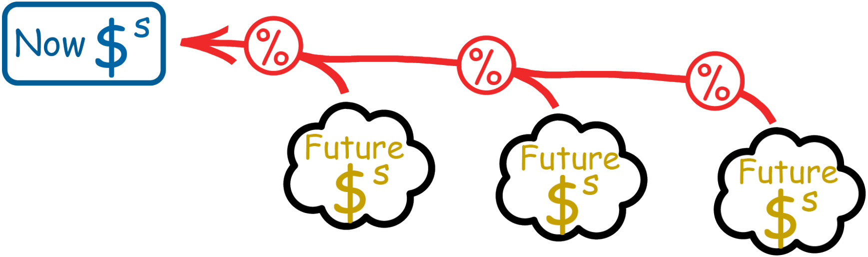


https://www.mathsisfun.com/money/net-present-value.html

$$\frac{500}{((1+\frac{.04}{12})^{168})} =$$

How much should we put in now as a lump sum if we want the future value (FV) to be \$500 after 14 years of an account paying 1% compounded monthly (i.e. what is the present value (PV) of the account)?

$$500 = PV(1 + \tfrac{.04}{12})^{14 \cdot 12} = PV(1 + \tfrac{.04}{12})^{168}$$



https://www.mathsisfun.com/money/net-present-value.html

$$\frac{500}{((1 + \frac{.04}{12})^{168})} = 434.70$$

Futurama: A Fishful of Dollars

Bank Teller: *Ok. You had a balance of 93 cents... and at an average of two and a quarter percent interest [compounded annually] over a period of 1000 years, that comes to...*

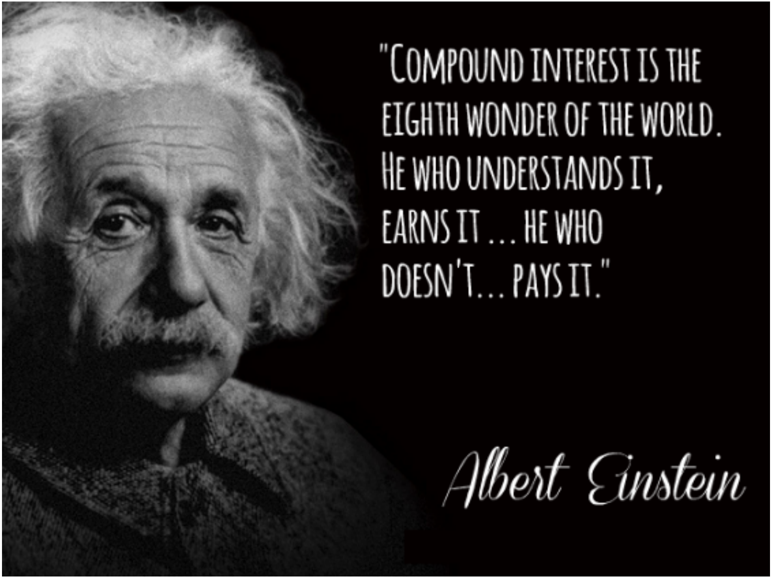
Futurama: A Fishful of Dollars

Bank Teller: *Ok. You had a balance of 93 cents... and at an average of two and a quarter percent interest [compounded annually] over a period of 1000 years, that comes to...*

4.3 billion dollars

Futurama: A Fishful of Dollars

$.93(1 + .0225)^{1000} \approx \$4,283,508,449.71$



1947 photograph of Einstein by Orren Jack Turner, public domain