Application of Algebra: Periodic Payment

$100 every month for 25 years, compounded monthly at 5%?
deposit money at the end of each compounding period

http://image.naldzgraphics.net/2012/10/8-periodic-pay.jpg

- future value of each payment via lump sum:
  1st payment of $100 grows to

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- future value of each payment via lump sum:
  1st payment of \$100 grows to $100(1+\frac{.05}{12})^{299}$
  2nd payment of \$100 grows to

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- future value of each payment via lump sum:
  1st payment of \$100 grows to $100(1 + \frac{.05}{12})^{299}$
  2nd payment of \$100 grows to $100(1 + \frac{.05}{12})^{298}$
  ...
  299th payment of \$100 grows to

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  1st payment of $100 grows to $100(1+\frac{.05}{12})^{299}$
  2nd payment of $100 grows to $100(1+\frac{.05}{12})^{298}$
  ...
  299th payment of $100 grows to $100(1+\frac{.05}{12})^{1}$
  300th payment of $100 grows to $100(1+\frac{.05}{12})^{0} = 100$

Periodic Payment: It's all about the Benjamins

\$100 every month for 25 years, compounded monthly at 5%?

- total savings plus interest (FV for short on these slides)

  sum the future value of each payment:

$$100(1+\frac{.05}{12})^{299}+100(1+\frac{.05}{12})^{298}+\ldots+100(1+\frac{.05}{12})^{1}+100$$

"an investment in knowledge pays the best interest"

Image Credit: Dan Rosandich

- FV=$100(1+\frac{.05}{12})^{299}+100(1+\frac{.05}{12})^{298}+\ldots+100(1+\frac{.05}{12})^{1}+100$
- too many terms—as many as compounding periods!
- shift our view via algebraic argumentation as in the reading
- FV(1+rate) = FV$(1+\frac{.05}{12})$...

Mat Moore, 2008

1. FV=$100(1+\frac{.05}{12})^{299}+100(1+\frac{.05}{12})^{298}+\ldots+100(1+\frac{.05}{12})^{1}+100$
2. FV(1+rate) = FV$(1+\frac{.05}{12})$ =

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3. distribute

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   FV$(1+\frac{.05}{12})-$ FV = $100(1+\frac{.05}{12})^{300}-100$

7. a bit more algebra: factor 100 and FV and solve for FV:

   FV $(1+\frac{.05}{12}-1)=100((1+\frac{.05}{12})^{300}-1)$

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7. a bit more algebra: factor 100 and FV and solve for FV:

   FV $(1+\frac{.05}{12}-1)=100((1+\frac{.05}{12})^{300}-1)$

$$\text{total savings + interest}=\frac{100((1+\frac{.05}{12})^{300}-1)}{\frac{.05}{12}}$$

periodic payment: total savings + interest

$$= \frac{100((1+\frac{.05}{12})^{300}-1)}{\frac{.05}{12}}$$

- sum the future value of each payment
- too many terms—as many as compounding periods!
- shift our view via transforming it by a common piece (1+rate) and then we combined the shifted equation with the original (subtraction). The overlap cancelled to give us a manageable formula

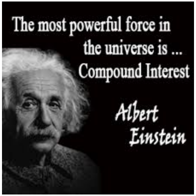

Image credits: Inkley, http://investorhorizon.com/wp-content/uploads/2015/02/ci7.jpg

Periodic Payment and Total Interest

periodic payment

\$100 every month for 25 years, compounded monthly at 5%?

total savings + interest =

$$\frac{\text{regular payment}((1+\text{periodic rate})^{\#\text{ times compounded}}-1)}{\text{periodic rate}}$$

$$\frac{100((1+\frac{.05}{12})^{300}-1)}{\frac{.05}{12}}$$

$$\frac{100((1+\frac{.05}{12})^{300}-1)}{(\frac{.05}{12})}$$

= \$59,550.97

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= \$59,550.97

total interest? total savings plus interest - amount we put in

$= \$59,550.97 - 100 \times 12 \times 25 = \$29,550.97$

$= \$59,550.97 - 100 \times 300 = \$29,550.97$

What algebraic operations did we use to derive the periodic payment/annuity formula?

a) multiplication, distribute, factor
b) rule for powers—add the exponents
c) subtraction
d) all of the above

Mat Moore, 2008

Periodic Payment: It's all about the Benjamins

\$100 every month for 25 years, compounded monthly at 5%?

$$\text{total} = \frac{100((1+\frac{.05}{12})^{300}-1)}{\frac{.05}{12}} = \$59550.97$$

$$\text{interest} = \$59550.97 - 100 \times 12 \times 25 = \$29550.97$$

\$100 every year for 25 years, compounded annually at 5%?

$$\text{total} = \frac{100((1+.05)^{25}-1)}{.05} = \$4772.71$$

$$\text{interest} = 4772.71 - 100 \times 25 = \$2272.71$$

Lump and Periodic Payments

Taylor deposits $100 a month into an account paying 1.75% compounded monthly for 7 years then changes the deposit to $175 each month for 5 more years at the same rate. What will the total savings plus interest be after the entire 12 years? It might help to think of the $100 deposits in one account and the $175 in another that we add together at the end.

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\$100 for first 7 years: $= \dfrac{100((1+\frac{.0175}{12})^{(12\cdot 7)}-1)}{(\frac{.0175}{12})} = \$8929.25$

\$8929.25 for last 5 years: $8929.25(1+\frac{.0175}{12})^{(12\cdot 5)} = \$9745.14$

\$175 for last 5 years: $= \dfrac{175((1+\frac{.0175}{12})^{(12\cdot 5)}-1)}{(\frac{.0175}{12})} = \$10964.72$

total is 9745.14+10964.72=\$20709.86

What is the total interest after the entire 12 years?

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total is 9745.14+10964.72=$20709.86

What is the total interest after the entire 12 years?
20709.86-100*7*12 -175*5*12 = $1809.86

Lump or Periodic Payments

On Sep. 29th, former employees... must decide whether to take a lump-sum payout on their pension or take a monthly pension check in the future. The decision should not be taken lightly. Depending on which way the market winds blow and what assumptions you use, this could be a $1 million decision.

www.huffingtonpost.com/mike-branch-cfp/should-you-say-yes-to-you_b_5889992.html

www.thejacobsfinancialgroup.com/choosing-between-a-lump-sum-and-periodic-payments/

America's Got Talent: "The prize, which totals $1,000,000, is payable in a financial annuity over forty years, or the contestant may choose to receive the present cash value of such annuity."

https://jborden.com/wp-content/uploads/2016/08/decisions.jpg

For the lottery, what would you take?

a) lump sum option
b) periodic payment option
c) neither—I would refuse the winnings

BUSINESS

What Becomes of Lottery Winners?

Millions are buying Powerball tickets assuming that winning will bring them a prosperous, work-free life, but research suggests they shouldn't be so certain.

BOURREE LAM JAN 12, 2016

STEPHANIE KEITH / REUTERS

https://www.theatlantic.com/business/archive/2016/01/lottery-winners-research/423543/

Lump Versus Periodic Payment

- lump sum
  total $= \text{lump}(1+r)^n$
  total interest= total $-$ lump
  one-time-principal deposit
  or an account that converts over to lump sum
  after no new additional principal additions
- periodic payment
  total = $\dfrac{\text{PMT}((1+r)^n - 1)}{r}$
  total interest= total $-PMT \times n$
  repeated deposit of new principal money for savings

What is the total savings plus interest when $25 is deposited into an account every month for 8 months at 1% compounded monthly?

a) $25(1+\frac{.01}{12})^{8\times 12}$

b) $\dfrac{25((1+\frac{.01}{12})^{8\times 12}-1)}{\frac{.01}{12}}$

c) $\dfrac{25((1+\frac{.01}{8})^{8}-1)}{\frac{.01}{8}}$

d) $\dfrac{25((1+\frac{.01}{12})^{8}-1)}{\frac{.01}{12}}$

e) none of the above

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total: $200.58 total interest: $200.58-25\times 8=\$.58$

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For a) and b), write scenarios that represent each

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e) none of the above

total: \$200.58 total interest: $200.58 - 25 \times 8 = \$.58$

For a) and b), write scenarios that represent each

a) \$25 now and left for 8 years at 1% compounded monthly

total: 27.08 interest: $27.08 - 25 = \$2.08$

b) \$25 each month for 8 years at 1% compounded monthly

total: 2497.53 interest: $2497.53 - 25 \times 8 \times 12 = \$97.53$