

Andrew Wiles, 1953-present

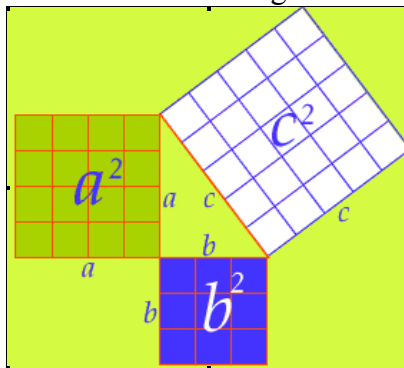
The Concept of Proof: Checking all the Cases and Constructivism

Question 1:

How many solutions does the Pythagorean equation $a^2 + b^2 = c^2$ have?

- a) none
- b) 1
- c) 3
- d) 15
- e) infinitely many

It would be hard to prove why the Pythagorean Theorem holds just by using algebra. Instead, we will move the problem to the world of geometry. Algebraic geometers use this process all the time to solve problems in algebra or geometry. They work in whichever realm is easiest, and then they translate the problem and solution back to the other world. An area argument:



<http://www.pbs.org/wgbh/nova/proof/puzzle/theorem.html>

Mathematician David Henderson explains that:

An ideal proof...

is a communication – the nature of this communication, of course, depends on the community to which one is communicating and is thus in part a social phenomenon.

is convincing -- a proof "works" when it convinces others in all cases.

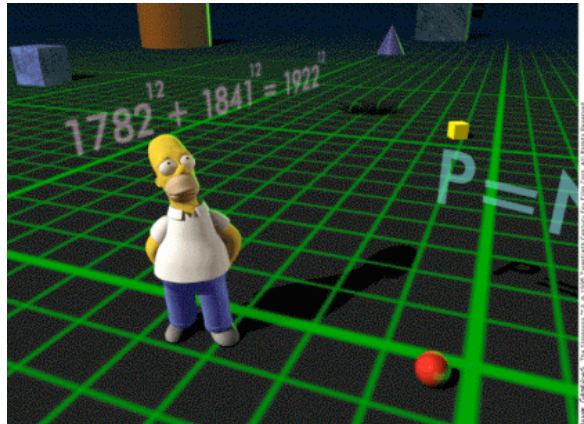
answers -- Why? -- The proof should explain something that the hearer of the proof wants to have explained. I think most people in mathematics have had the experience of logically following a proof step by step but are still dissatisfied because it did not answer questions of the sort: "Why is it true?" "Where did it come from?" "How did you see it?" "What does it mean?"

Question 2: Find at least one portion of David Henderson's definition of an ideal proof (*a convincing communication that answers --Why?*) that the above area exploration fails. Prepare to share your thought process

- a) communication
- b) convincing
- c) answers why
- d) more than one

Question 3: Fermat said that you could not find any non-zero whole number solutions to the equation $a^n + b^n = c^n$ when $n > 2$. In a mathematical proof you have to write down a line of reasoning demonstrating why or why not. If the proof is rigorous, then nobody can ever prove it wrong. Can a computer check that there are no solutions?

- a) yes
- b) no



$1782^{12} + 1841^{12} = 1922^{12}$ Flies Past 3D Homer in *The Simpsons: Treehouse of Horror VI*

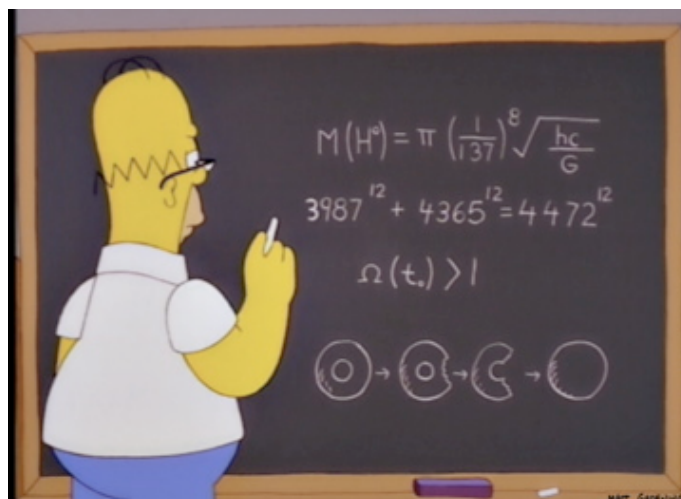
Question 4: Is the right hand side of the equation even or odd? The left hand side? So is the equation true?

- a) yes
- b) no

Question 5: Use Fermat's Last Theorem (which Andrew Wiles proved was true) to explain why the equation cannot hold.

Question 6: I decide to calculate the 12th root of $1782^{12} + 1841^{12}$. On my TI-83 Plus calculator. I type $1782^{12} + 1841^{12}$ ENTER and obtain 2.541210259 E 39. I then take the 12th root by typing $^{1/12}$ ENTER and obtain 1922, showing the equation does hold. This really did happen on my calculator. Try this on your calculator. What happens on your calculator?

- a) My calculator also shows 1922
- b) My calculator does not show 1922



$3987^{12} + 4365^{12} = 4472^{12}$ in *The Simpsons: Wizard of Evergreen Terrace*

Question 7: The first reference from *The Simpsons* appeared before Andrew Wiles had completed his proof, but we can still explain why they are false, even without Fermat's Last Theorem. Try an argument similar to the one we used in Question 4: what is the last digit of 3987^{12} ? Of 4365^{12} ? Of 4472^{12} ?

Question 8: Are any of the numbers divisible by 3? Is one or both sides of the equation divisible by 3?

Andrew Wiles proved Fermat's Last Theorem by showing that Taniyama-Shimura is true. At first glance, it appeared that Taniyama-Shimura was unrelated to Fermat's Last Theorem. Taniyama-Shimura said that all elliptic curves (donuts) are modular forms (symmetries), and gave a dictionary in order to translate problems, intuition, equations and proofs between these two worlds. Even though Taniyama-Shimura had not yet been proven, many mathematical ideas came to depend on it. The Epsilon-Conjecture related Taniyama-Shimura to Fermat's Last Theorem. It stated that if Fermat's Last Theorem was false and there was a non-zero whole number solution to Fermat's Last Theorem, then this solution would be so weird that one could use it to find an elliptic curve that was not modular, and so Taniyama-Shimura would also be false. Andrew Wiles then indirectly proved Fermat's Last Theorem by showing that Taniyama-Shimura is true – he came up with a way to separate the infinitely many elliptic curves into packets to show they were all modular.

Constructive Proofs:

Although certain individuals — most notably Kronecker — had expressed disapproval of the “idealistic”, nonconstructive methods used by some of their nineteenth century contemporaries, it is in the polemical writings of L.E.J. Brouwer (1881-1966), beginning with his Amsterdam doctoral thesis (Brouwer 1907) and continuing over the next forty-seven years, that the foundations of a precise, systematic approach to constructive mathematics were laid. In Brouwer's philosophy, known as intuitionism, mathematics is a free creation of the human mind, and an object exists if and only if it can be (mentally) constructed.

<http://plato.stanford.edu/entries/mathematics-constructive/>

Question 9: Explain in what way Wiles' proof might not be considered constructive

The Simpsons TM and copyright [Twentieth Century Fox](#) and its related companies. This worksheet is for educational use.