## Student, House, and Car Loans

- words with mort are often deadly \& among them are: mortgage="death pledge" amortize a debt=to "kill the debt"
- events described all actually happened and the same language is purposely used
- when our own future is at stake, most of us want to use every approach we can-many cases require the critical and creative analysis of a variety of interpretations in order to fully consider the implications
- notice the loan rates are higher than the savings rates, factoring in risk


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installment payment $=$ loan $r \frac{1}{1-(1+r)^{-n}}=\frac{\text { loan } r}{1-(1+r)^{-n}}$

## Loan Payments and Amortization

loan amount $r$
$\frac{\text { loan amount } r}{1-(1+r)^{-n}}=$ loan payment

- total paid= payment $\times$ \# times compounded - overpayment
- total interest = total paid - loan


## Loan Payments and Amortization

Ioan amount $r$
$\frac{1-(1+r)^{-n}}{1-(l o a n ~ p a y m e n t}$

- total paid= payment $\times$ \# times compounded - overpayment
- total interest = total paid - loan
- interest each period on a loan is computed just as in savings:
account balance $\times$ periodic rate but now we pay it back rather than earn it


## Congratulations—Now Feed Me Your Loan Payments!


https://www.brookings.edu/blog/up-front/2020/04/16/whats-the-government-done-to-relieve-student-loan-borrowers-

## Dear Sarah J Greenwald

At this time you have a choice of repayment terms for your student loan $\$ 4795.00$ at $8 \%$ compounded monthly:

Graduated Repayment Plan \# PMTS PMT AMT
$\begin{array}{ll}24 & 34.05 \\ 24 & 44.79\end{array}$
$24 \quad 58.92$
$24 \quad 77.50$
$23 \quad 101.94$
196.92


Level Payment Plan \# PMTS PMT AMT 11958.18
$1 \quad 57.55$
total $\$ 6980.97$

## Student Loan

student loan $\$ 4795.00$ at $8 \%$ compounded monthly: loan $r$ installment payment $=\frac{\text { loan } r}{1-(1+r)^{-n}}$

$$
=\frac{4795 \frac{.08}{12}}{\left(1-\left(1+\frac{.08}{12}\right)^{-120}\right)}=
$$

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student loan \$4795.00 at 8\% compounded monthly: installment payment $=\frac{\text { loan } r}{1-(1+r)}$

$$
=\frac{4795 \frac{.08}{12}}{\left(1-\left(1+\frac{.08}{12}\right)^{-120}\right)}=58.176581 \ldots
$$

amortization table for level payment plan
mo.
$58.18 \quad 4795 \frac{.08}{12}=31.97$ balance $\times$ periodic rate
principal paid loan balance
58.18-31.97 4795-26.21=4768.79
payment-int balance-principal

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$=26.39=4742.4$

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mo.
$2 \quad \begin{gathered}58.18 \quad 4768.79 \cdot \frac{08}{12}=31.79 \quad 58.18-31.7 \\ =26.39\end{gathered}$ total paid: $58.18 \times 120-.63=6980.97$
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$=26.39 \quad=4742.4$
last months payment: $58.18-.63=57.55$ total paid: $58.18 \times 120-.63=6980.97$
total interest: $6980.97-4795$


- Write down the loan payment formula with numbers filled in for an 84212 loan for 30 years compounded at $6.75 \%$ monthly.
- Solve for the monthly payment. loan amount $r$ $\frac{1-(1+r)^{-n}}{1-\text { loan payment }}$
- Calculate the first two rows of the amortization table. month payment interest paid principal paid loan balance
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- Solve for the monthly payment. Ioan amount $r$ $\frac{1-(1+r)^{-n}}{1-(\text { loan payment }}$
- Calculate the first two rows of the amortization table. month payment interest paid principal paid loan balance

|  | A | B | C | D | E |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | condo cost | 105265 | monthly rate | 0.005625 |  |
| 2 | loan amt | 84212 | 1st years total interest | \$5,656.88 |  |
| 3 | payment | \$546.20 | total interest 30 years | \$112,419.07 |  |
| 4 |  |  |  |  |  |
| 5 | month \# | End of Month Payment | Interest Paid that Month | Principal Paid that Month | Loan Balance |
| 6 | 1 | \$546.20 | 473.6925 | \$72.50 | \$84,139.50 |
| 7 | 2 | \$546.20 | \$473.28 | \$72.91 | \$84,066.58 |

