## Loan Payments and Amortization

$$
\text { payment }=\frac{\text { loan amount } r}{1-(1+r)^{-n}}=\frac{14500 \cdot \frac{12}{12}}{\left(1-\left(1+\frac{.12}{12}\right)^{(-12 \times 4)}\right)}=\$ 381.84
$$

month Payment Interest Paid Principal Paid Loan Balance

| 1 | 381.84 | $\$ 145$ | $\$ 236.84$ | $\$ 14,263.16$ |
| :--- | :--- | :--- | :--- | :--- |
|  |  | $14500 \cdot \frac{12}{12}$ | $381.84-145$ | $14500-236.84$ |
| 2 | 381.84 | $\$ 142.63$ | $\$ 239.21$ | $\$ 14,023.95$ |
|  |  | $14263.16 \cdot \frac{12}{12}$ | $381.84-142.63$ | $14263.16-239.21$ |
| 3 | 381.84 | $\$ 140.24$ | $\$ 241.60$ | $\$ 13,782.35$ |
|  |  | $14023.95 \frac{.12}{12}$ | $381.84-140.24$ | $14023.95-241.60$ |

- total paid $=381.84 \times 12 \times 4$ - overpayment
- total interest $=$ total paid - loan $=381.84 \times 12 \times 4-14500$



## Loan Payments

lender earns what it could elsewhere, we pay in installments: lump sum of loan = periodic payment of our monthly payment Ioan amount $(1+r)^{n}=\frac{\text { monthly payment }\left((1+r)^{n}-1\right)}{r}$
$r=$ periodic rate (like $\frac{.05}{12}$ )
$n=$ \# times compounded (like 120 or 360 )
(1) loan amount $r \frac{(1+r)^{n}}{(1+r)^{n}-1}=$ loan payment
(2) reduce further using $x=(1+r)^{n}$

$$
\frac{(1+r)^{n}}{(1+r)^{n-1}}=\frac{x}{x-1}=\frac{x}{x-1} \frac{\frac{1}{x}}{\frac{1}{x}}=\frac{1}{x \frac{1}{x}-1 \frac{1}{x}}=\frac{1}{1-\frac{1}{x}}=\frac{1}{1-(1+r)^{-n}}
$$

(3) sub back in

$$
\frac{\text { loan amount } r}{1-(1+r)^{-n}}=\text { loan payment }
$$

