

handwrite Benjamin Franklin's financial legacy

Dr. Sarah's 1010: Introduction to Mathematics

submission instructions: Must be handwritten, preferably on this handout, and collated into one multipage PDF for submission in the ASULearn assignment.

goals:

- Explore applications of algebra in everyday life, investigate real-world data and interpret.
- Utilize technology to adapt and use mathematical formulas that include cell referencing to answer real-world questions and interpret results.
- Communicate quantitative information using a variety of representations, including numerical, algebraic, and tables, in written documents.

Even the best-laid plans...

There have been numerous presidents and founders who have connections to mathematics. James Garfield, who served as the 20th president, created a proof of the Pythagorean Theorem. Recent presidents have highlighted the importance of mathematics and science in global competitiveness. Benjamin Franklin is considered a founding father and he left a financial legacy:

“...I wish to be useful even after my Death, if possible, in forming and advancing other[s]... To this end I devote Two thousand Pounds Sterling... one thousand thereof to the Inhabitants of the Town of Boston in Massachusetts, and the other thousand to the Inhabitants of the City of Philadelphia, in Trust and for the Uses, Interests and Purposes hereinafter mentioned.....”

Even though the fund charged borrowers 5% interest (as Franklin had planned on earning), it was not always possible to find as many borrowers as Franklin had planned and there were other problems as well since some of the borrowers did not pay back their loans. The *average earned rate* is the weighted average of all the rates each part of the money actually earns.

1. Hypothetical Situation: If the fund lent out half of its money at 5% to borrowers who all paid back their loans plus interest, but if the fund could not find any borrowers for the other half of the money (i.e. 0%), what would the average earned rate of the fund be (take the weighted average—half at 5% and half at 0%)?

Circle one: 0% 2.5% 5%

2. The fund lent out some of its money to borrowers who didn't repay anything, which would bring the average earned rate of the entire fund down. What would the earned rate on just the lost money be—the lost principal plus the lost interest on it?

Circle one: negative 0% positive

The Fund in Boston

In real life, in January 1894, at the end of 100 years from the inception of the Franklin gift, because of these types of problems, the Boston fund had grown from \$4444.44 dollars (the equivalent of \$1000 pounds) to \$391,000 dollars. (Note that this is less than the \$582,221.64 (the equivalent of 131,000 pounds) that Franklin had imagined, because even though the fund charged 5%, the problems reduced the earnings.) We'll use Excel to solve the lump sum equation for the unknown rate.

Why lump sum? Benjamin Franklin only put in a one-time infusion of money, so lump sum is appropriate as the rest of the money during the first 100 years comes in as what was borrowed plus interest. This exactly matches the scenario we used when we derived the formula: We obtained the formula for lump

sum using the total from the year before to calculate the principal and interest for the next year. This process works fine, but is too difficult to use when the number of years is large. So we looked for a way to obtain a simplified formula. We looked for the commonality and recognized the repeated appearance of $(1+rate)$ after factoring, which represents one lump principal with only new interest coming in.

The first hundred years in Boston

- In this context lump sum is $391,000 = 4444.44(1+aer)^{100}$ because 4444.44 was the principal at the beginning of the 100 years, and 391,000 was the ending value at the end of the 100 years, just before the fund was modified. Here aer stands for average earned rate.
- Download the Excel file **benf1.xls** from the ASULearn assignment. You will also see some other items there that you will be using later. On your computer, the Excel file may come up automatically or you will open it yourself. You will then see a chart that is partly filled in with descriptors.
- In the C2 box of your downloaded Excel file, type (and don't forget the equals sign that always comes before Excel equations!):

$$=4444.44*(1+D2)^{100}$$

and then hit return. Be sure that the cell reads 4444.44.

- Click again on C2.
- We want Excel to solve for the average earned interest rate that will result in 391000 so find Goal Seek in your Excel program. It may be under Data/What-If Analysis/Goal Seek or Tools/Goal Seek or elsewhere.
- In the Goal Seek box,

Set cell: should already read C2

To value: Put 391000 in the to value slot, which is the total savings plus interest the fund grew to at the end of the first 100 years.

By changing cell: Type D2 in the changing cell box to have Excel change the rate to account for the growth (average earned rate)

Goal Seek will find a solution, so then click on OK. Notice that the average earned interest rate solution is in box D2.

3. Aside from the fact that I converted the rate to a percentage, does your Excel formula and average earned rate work match columns A through D in the first row of numbers in the Table below?

circle one: yes no (if not, redo) Notice that I added two columns that aren't in the table in Excel. Review these before you continue to the next page (skip the other rows for now).

A	B	C	D		
City	Time	Excel formula	aer as a %	Goal Seek amount	Lump sum equation with rate left as unknown
Boston	100	$=4444.44*(1+D2)^{100}$	4.5787863%	\$391,000	$391,000 = 4444.44(1+aer)^{100}$
Boston question #4 SEE BELOW	200				
Philly question #5	100				
Philly question #6	200				

The second hundred years in Boston

As per Ben Franklin's wishes, at the end of the first hundred years, a portion of the earnings from the first hundred years went back into the fund to be loaned out, while the remainder was given to the cities. In Boston, \$100,000 of the \$391,000 was reinvested at the end of the first hundred years, so we start with that as the new principal. By lending money to borrowers at 5% interest, the fund grew to 5 million dollars (\$5,000,000) at the end of the second hundred years. Since the lump sum principal has been modified from the original, we must start a new lump sum calculation for the second hundred years, with the new principal amount. Here is both sides of the lump sum formula with the numbers filled in for the second 100 year period: $5,000,000 = 100,000(1 + a)^{100}$. Notice that the exponent is 100, not 200, because we are looking at growth from the end of the first hundred years to the end of the second (i.e. 100 years of growth).

4. Use Excel to solve for the average earned rate of the fund (the answer is not 5% since the fund faced similar problems as were faced during the first hundred years). Be careful that you set up the lump sum formula in the correct box (C3), refer to (D3) for the unknown rate, and that you use Goal Seek on the correct box (C3). Make sure your average rate is approximately 3.98% and then fill in the next row of the table back in question 3.

The Fund in Philly

The first hundred years in Philly

5. At the end of first 100 years the Philadelphia fund had grown from \$4444.44 dollars to only \$172,350 dollars. In the above table, fill in both the left and right hand sides of the lump sum formula with these numbers filled in (but leave the earned rate of the fund as a variable). Next, solve for the average earned interest rate responsible for this Philadelphia growth in your Excel sheet in boxes C4 and D4.

The second hundred years in Philly

6. In Philadelphia, \$39,274 was reinvested at the end of the first hundred years and the fund grew to \$2,256,952.05 by the end of the second hundred years. In the above table, fill in both the left and right hand sides of the lump sum formula with the numbers filled in (but leave the earned rate of the fund as a variable). Next, solve for the average earned rate of the fund in your Excel sheet in boxes C5 and D5. Make sure your average rate is approximately 4.13%.
7. Fill in the relevant rows of the table back in question 3 with all of Philly's info.
8. In *The Heart of Mathematics* you read about "The difference that one percentage point makes" for a hypothetical situation involving Adam and Eve. We see similarities here. During the first hundred years, Boston partially gave up on Franklin's plan and invested in savings accounts and a life insurance company rather than loaning to individuals in order to help them.
 - (a) What is the approximate difference in Boston and Philadelphia's average earned rates during the first 100 years? Subtract.
 - (b) Who had a higher average earned rate? Circle one: Boston Philly exactly the same
 - (c) What is the difference in the average earned rates during the second 100 years?
 - (d) Who had a higher average earned rate? Circle one: Boston Philly exactly the same

15. Which part of what actually happened in Boston was furthest from Franklin's plans/intentions/motivations?

16. Which part of what actually happened in Philly was closest to Franklin's plans/intentions/motivations?

17. Which part of what actually happened in Philly was furthest from Franklin's plans/intentions/motivations?

Decisions, Decisions

18. After 200 years, Boston and Philadelphia had to decide what to do with their respective earnings over the long run. What would you do with these funds for one of the cities—relate your answer to Benjamin Franklin's original goals and wishes (i.e. no keeping it for yourself!):
I wish to be useful even after my Death, if possible, in forming and advancing other[s]...
At the end of this second term, if no unfortunate accident has prevented the operation, the sum will be four millions and sixty-one thousand pounds sterling; of which I leave... to the disposition of the inhabitants... not presuming to carry my views farther.

19. Collate your handwritten responses, preferably on this handout, into one full size multipage PDF for submission in the ASULearn assignment.