<u>Question 1</u>: If we put in $100 now and leave it there for 25 years compounded monthly at 5% how much will we end up with? How much of that will be interest ($)?

<u>Question 2:</u> If we put in $100 each month for 25 years compounded monthly at 5% how much will we end up with? How much will be interest ($)?

Question 2:

Put in \$100/month *12 months/year * 25 years = \$30,000, so we will end up with \$30,000 + interest (\$) = ?

Not a lump sum because we put in money each month – start with what we know and try to use it to develop a new formula.

25 years from now:

Our 1st payment of \$100 will have grown to $100(1+\frac{.05}{12})^{299}$

Our 2nd payment of \$100 will have grown to $100(1+\frac{.05}{12})^{298}$

…

Our 299th payment of \$100 will have grown to $100(1+\frac{.05}{12})^{1}$

Our 300th payment of \$100 will have grown to $100(1+\frac{.05}{12})^{0} = 100$

Note: Banks set it up so that we deposit money at the end of each month, so the money gains interest 1 month less than we expect (for example, 299 = 25*12-1=300-1).

Equation 1: Total Savings Plus Interest =

$$100(1+\frac{.05}{12})^{299} + 100(1+\frac{.05}{12})^{298} + 100(1+\frac{.05}{12})^{297} + \ldots + 100(1+\frac{.05}{12})^{1} + 100$$

Each term in Equation 1 is the future value of one payment we make using the lump sum formula. But, Equation 1 is too complicated to use for many months or years. We'll step back and shift our view of it. We'll transform it by a common piece and then combine the shifted equation with the original. The overlap will cancel so that we can obtain a general formula to use. This is not just a trick – it is a different way of looking at things.

Equation 1: Total Savings Plus Interest =

$$100(1+\tfrac{.05}{12})^{299} + 100(1+\tfrac{.05}{12})^{298} + 100(1+\tfrac{.05}{12})^{297} + \ldots + 100(1+\tfrac{.05}{12})^{1} + 100$$

Each term in Equation 1 is the future value of one payment we make using the lump sum formula. But, Equation 1 is too complicated to use for many months or years. We'll step back and shift our view of it. We'll transform it by a common piece and then combine the shifted equation with the original. The overlap will cancel so that we can obtain a general formula to use. This is not just a trick – it is a different way of looking at things.

Shifted Equation:

Equation 1 * $(1+\tfrac{.05}{12})$ = Total Savings Plus Interest* $(1+\tfrac{.05}{12})$ =

$$\left[100(1+\tfrac{.05}{12})^{299} + 100(1+\tfrac{.05}{12})^{298} + \ldots + 100(1+\tfrac{.05}{12})^{1} + 100\right] * (1+\tfrac{.05}{12}) =$$

Now distribute the $(1+\tfrac{.05}{12})$ term inside the square brackets

$$=[100(1+\tfrac{.05}{12})^{299}(1+\tfrac{.05}{12}) + \ldots + 100(1+\tfrac{.05}{12})^{1}(1+\tfrac{.05}{12}) + 100(1+\tfrac{.05}{12})]$$

We combine the $(1+\tfrac{.05}{12})$ terms using exponent rules blah^{n} blah = blah^{n+1}

$$=[100(1+\tfrac{.05}{12})^{300} + 100(1+\tfrac{.05}{12})^{299} + \ldots + 100(1+\tfrac{.05}{12})^{2} + 100(1+\tfrac{.05}{12})]$$

=Total Savings Plus Interest* $(1+\tfrac{.05}{12})$

Each term in Equation 1 is the future value of one payment we make using the lump sum formula. But, Equation 1 is too complicated to use for many months or years. We'll step back and shift our view of it. We'll transform it by a common piece and then <u>combine the shifted equation with the original. The overlap will cancel so that we can obtain a general formula to use. This is not just a trick – it is a different way of looking at things.</u>

<u>Shifted Equation – Equation 1:</u>
<u>The left hand sides:</u>

$$\text{Total Savings Plus Interest}^* \left(1+\frac{.05}{12}\right) - \text{Total Savings Plus Interest}$$

<u>The right hand sides:</u>

$$\left[100\left(1+\frac{.05}{12}\right)^{300} + 100\left(1+\frac{.05}{12}\right)^{299} + \ldots + 100\left(1+\frac{.05}{12}\right)^{2} + 100\left(1+\frac{.05}{12}\right)\right]$$
$$- \left[100\left(1+\frac{.05}{12}\right)^{299} + 100\left(1+\frac{.05}{12}\right)^{298} + 100\left(1+\frac{.05}{12}\right)^{297} + \ldots + 100\left(1+\frac{.05}{12}\right)^{1} + 100\right]$$

Cancel out common terms

$$= 100\left(1+\frac{.05}{12}\right)^{300} - 100$$

We now have <u>Shifted Equation – Equation 1</u> =

$$\text{Total Savings Plus Interest}^* \left(1+\frac{.05}{12}\right) - \text{Total Savings Plus Interest}$$
$$= 100\left(1+\frac{.05}{12}\right)^{300} - 100$$

We want to solve for Total Savings Plus Interest. Factor this out and solve!