1010 Geometry of the Earth and Universe T/Th Questions

Here are portions of questions from class to help you with your notes or later practice. The wording and ordering may change and we may not have time to cover all of them. Here we actively practice concepts, computational strategies, critical & creative thinking, and communication. Making mistakes is integral to the learning process and enriches our understanding as we extend content and clear up misconceptions.

- **Think** about a possible answer(s) on your own.
- **Pair up**: discuss your thoughts in a group. We may reorganize groups at times.
- Prepare to **share** from your group's discussion. This may take the form of an assertion, question, definition, example, or other connection. It could be something you tried and rejected.
- May be a lag at times—use this to **review** related concepts and examples, and **add** to your notes, or get to know your neighbors.

Appalachian's General Education Program prepares students to employ various modes of communication. Successful communicators interact effectively with people of both similar and different experiences and values and in this class you will practice oral and written communication during class by interacting with various peers and me.

geometry intro

- How could we know that the earth is round without using modern technology from the 20th or 21st centuries?
- What is a flat angle sum and why/how do we know?
- What is dimension?
- What is parallel?
- *Shape of the World* & *Seeing is Believing* from *Life by the Numbers* Video

  Mathematics. We'd all be lost without it. Literally. Without mathematics we'd have no maps, no globes, no idea what the world really looks like. From Renaissance paintings to major motion pictures, the world of mathematics plays a pivotal role.

  Goals:

  Explore applications of geometry in everyday life.

  Identify geometric ideas/connections within local and global geographical regions.

  Communicate geometric information in written documents.

  The people in the video:

  American Actor & Director (narrator here): **Danny Glover**

  British Artist and Mapmaker: **Nigel Holmes**

  American Art Historian: **Sam Edgerton**

- What are the ways that the people talk about, explore or represent the earth throughout the video (what do they say about it, what are the analogies, visual representations...)? List as many as possible.

- What does geometry mean?

- What are the challenges with maps and geometric representations?

- What are the advantages of perspective geometry?

- Identify an instance from the video of geometric ideas/connections within local (small-scale) and global (large-scale) geographical regions. List them as follows:

  local (small-scale) region: geometric ideas/connections:

  global (large-scale) region: geometric ideas/connections:

- How does the industrial revolution connect?

- What were the techniques people employed to answer the question: Where is North?

- What is the most compelling argument (to you) about ways we could know that the earth was round without modern technology?

- In Eratosthenes' experiments he found the light ray at Alexandria made an angle of $7.2°$. Select a different angle that is between $6.2°$ and $8.2°$ (i.e. $7.2° \pm 1$, like say there was a margin of error in experimentation) and list what angle you selected. Set up the ratios, still using the 5000 stadia between the cities, and solve for the circumference. Compute the difference between the circumferences.

2D universes

- Think of a cartoon, anime or video game or character. What is the name of the cartoon, anime or video game or character? Apply our definition of dimension to their world (degrees of freedom of movement in space or efficient algebraic coordinates). What dimension do they live in? Explain how can you tell?

- What is straight on a curved surface?

- Locally, how do we know if we are on a curved space or flat Euclidean space?

- Escher's Representation of Hyperbolic Geometry/Saddle Geometry

Circle Limit 4: Heaven and Hell by M.C. Escher, 1960

- Pick an angel and label the tip of the angel's feet, and the two tips of her wings.
- We can figure out the angle at each of these three points by taking 360° and dividing by the total number of angels and demons that meet at that point, since each creature is supposed to take an equal amount of angle, fitting into 360°. For example, first check that at the feet there are 6 angels and demons coming into that point. So the angle at that point is $\frac{360^\circ}{6} = ?$
- A different number of creatures come in at each one the wingtips—how many creatures at a wingtip?
- What is the angle at each of her wingtips? Compute $\frac{360}{\text{number of creatures coming in}}^\circ =$
- What is the sum of the three angles (one at the feet and two at the wingtips)?
- Lines should preserve symmetry, so they should cut creatures in half (like mirror reflections) and go through the middle of the head, body, and feet. Use this idea to draw some "lines" in this space. Start by drawing mirror "lines" through the center of an angel that cuts her in half and continue these mirrors in both directions through other creatures. Draw at least five "lines" in the work, with at least 2 going through curvy angels and curvy demons.

- Which quote from Escher do you find most interesting?

  a) *The ideas... often bear witness to my amazement and wonder at the laws of nature which operate in the world around us... By keenly confronting the enigmas that surround us, and by considering and analyzing the observations that I had made, I ended up in the domain of mathematics* [The Graphic Work, 1954].

  b) *At first I had no idea at all of the possibility of building up my figures. I did not know any "ground rules" and tried, almost without knowing what I was doing, to fit together congruent shapes that I attempted to give the form of animals. Gradually, designing new motifs became easier as a result of my study of the literature on the subject, as far as this was possible for someone untrained in mathematics, and especially as a result of my putting forward my own layman's theory, which forced me to think through the possibilities. It remains an extremely absorbing activity* [Regular Division of the Plane, 1958].

  c) *The geometry of space translates to a reoccurring theme in my creations: the tessellation... had been considered solely in theory prior to me, some say. I diverged from traditional approaches, and chose instead to find solutions visually* [interview, January 17, 1971].

- Summarize the parables/analogies in *Mathematics The Most Misunderstood Subject* by Robert H. Lewis

- What does the author say about geometry?

- In flat Euclidean geometry of the infinite blackboard from high school, named for Euclid of Alexandria (~325 BCE–265 BCE), what is the sum of the angles in a triangle?

- In M.C. Escher's (1898-1972) *Circle Limit 4: Heaven and Hell* representation of hyperbolic geometry, what is the sum of the angles?

- How do we represent hyperbolic geometry?

- What are local to global connections?

- If we had another work from Escher where there were 5 creatures around one point, 6 creatures around another, and 8 creatures around the third, respond to **all of the following**

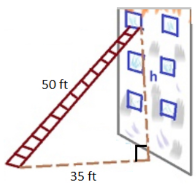
  a) What would the sum of the angles be?
  b) Would the space be hyperbolic?
  c) What would the angle sum tell you in terms of how curved the triangle is—is it very curvy?

- Are there any parallels in perspective drawing (projective geometry)?

- How many parallels are there to a line through a given point in flat Euclidean geometry?

- Is there more than one parallel in hyperbolic geometry?

- My husband is a professional musician who, in his spare time, volunteers for our local fire department and the rescue squad, as an EMT. If the firefighters have a 50 ft ladder and angle it towards the building, 35 ft away from the fire, how many feet high will it reach?



- Why is the Pythagorean theorem true?

- What happens to the Pythagorean theorem in hyperbolic geometry?

- Where is the Pythagorean theorem useful?

- What algebra arose to demonstrate the Pythagorean theorem based on the picture in the *Zhou Bi Suan Jing* or *Chou Pei Suan Ching*?

- In 4.6: The Shape of Reality? a saddle was an example of what kind of geometry?

- What are real-life applications of hyperbolic geometry?

- What would Spherius say to the idea of more than three dimensions existing, do you think?

- In a wraparound universe, we can head off on a path that feels straight to us and eventually come back around. Which are wraparound?

- How many dimensions does lineland have in *Flatland the Movie*?

- What could Arthur Square see at some point in time if a donut is dunked with the hole facing him? First think about what are cross sections in the 2D Flatland universe and then think about what would Arthur square could actually see (assume he can only see in the 2D universe)?

  a) 

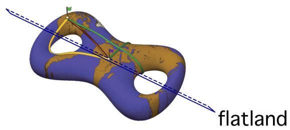
b) 

c) both

d) neither

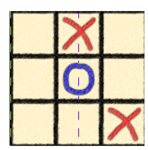
- A 2-holed donut arose in the 2D universe readings

  a) Sketch what the full cross section would be when a 2-holed donut passes through Flatland, in a view that includes the two holes.

  b) Sketch what Arthur square would see when a 2-holed donut passes by him in Flatland (assume that he can only see in the 2D universe), in a view that includes the two holes.

- Play a game of Klein bottle Tic-Tac-Toe: sketch the tiling view and label on the main board where o should go to block. Can o block a win from x?



earth

- A straight line on the surface of a sphere must curve from an extrinsic or external viewpoint, but intrinsically, say for example if we are living in Kansas, we can define what it means to feel like we are walking on a straight path. What is straight (intrinsically) on a sphere? Is the equator an intrinsically straight path? Is the non-equator latitude between Chicago and Rome an intrinsically straight path?

- For thousands of years, people argued about the necessity and validity of Euclid's Parallel Postulate. One form of this postulate is given as Playfair's Axiom: Through a given point, only one line can be drawn parallel to a given line. Is this true on the sphere?

- On the surface of a perfectly round beach ball, can the sum of angles of a spherical triangle (a curved triangle formed by three shortest distance paths on the surface of the sphere) ever be greater than 180 degrees? Why?

- Assume that we have a right-angled spherical triangular plot of land (a curved triangle formed by three shortest distance paths on the surface of the sphere that also contains a 90 degree angle) on the surface of a spherical globe between approximately the north pole, a point on the equator, and a point one-quarter away around the equator. Do the sides satisfy the Pythagorean Theorem? Why?

- Which did you find most compelling for why great circles are intrinsically straight and shortest distance paths?

- What arguments related to parallelism on the earth?

- Sketch a picture related to angle sum of the earth and summarize what it shows.

- What arguments related to the Pythagorean theorem on the earth?

Seeing is Believing/Shape of the World Video

Goals:

Explore applications of geometry in everyday life.
Identify geometric ideas/connections within local and global regions.
Identify the role of probability and chance in real world situations.
Communicate geometric information in written documents.

The people in the video: Actor Danny Glover: Narrator, Art Historian Linda Henderson, Artist Pablo Picasso, Mathematician Dufret, Mathematician Tom Banchoff, Artist Tony Robbin, Astrophysicist Rob Kirshner, Mathematician Robert Osserman, Physicist Albert Einstein, Mathematician George Riemann, Mathematician Jeff Weeks

- How do people in the video talk about, explore or represent higher dimensions (analogies, visual representations...)? For instance - Henderson uses x-rays and invisible realities as an analogy. Find as many as you can.

- What are ways that the researchers in the video model and explore the geometry of the universe? For example, Riemann and Einstein used a higher dimensional sphere. Also list any experiments in detail. Find as many as you can.

- Give one example from the video of a connection to the theme of local to global.

- Give one example from the video of a connection to the theme of the role of chance/probability.

universe

- Is our universe 3-dimensional or is it higher dimensional? Summarize diverse perspectives.

- Where does *The Heart of Mathematics* address higher dimensional spaces?

- Are there are finitely or infinitely many stars in the universe? Summarize diverse perspectives.

- We know that the shape of the earth is close to a round sphere. Could the universe be round too? Does it have any kind of shape? Summarize diverse perspectives.

- What sequence (over time) would we see if a hypersphere passed by us? Think about an analogue of a sphere passing by a 2-D creature's plane of existence, like when Spherius passed by Arthur Square in *Flatland*.

- Is the universe finite and wraparound? Summarize diverse perspectives.

- Does Carl Friedrich Gauss's mountain peak experiment to measure the angle sum prove that the universe is flat?

- Summarize diverse experiments related to the geometry of the universe including angle sum experiments, supernova experiments, and density experiments, including how the experiments relate, what they seemed to show, and critiques.

- Summarize the $\frac{1}{3000}$ part of the density experiments that Jeff Weeks mentioned.

- Summarize the "plus or minus" $\pm$ (margin of error for confidence interval) part of the density experiments that Jeff Weeks mentioned.

- Read the following interview. As you do so, circle or write down at least one or two items that you found interesting, disagreed with, had a question on, or wished had been done as related to the following:

  What influences led you to become a mathematician?

  Reading Abbot's *Flatland* during senior year in high school was a turning point. After two weeks of intense effort, almost in a flash I could finally "see" it. This glimpse into a new world made a huge impression on me. I think that's what hooked me on mathematics, and on geometry in particular. I love the connections between pure mathematics and the physical world. A real joy.

  Why did you become a mathematician?

  The combination of beauty and simplicity of exploring new worlds exceeds anything you'll find in everyday life. As part of the current research I've been working on the hypersphere. That's part of the joy of mathematics—you might struggle with something for months, but then when you "get it"...

  Are there any local to global issues or diversity issues in your experiences?

  In an international sense mathematics is extremely diverse, with people from a broad range of cultures sharing a common mathematical culture. Domestically, though, mathematics seems to be in decline, regardless of race or gender. The US now relies overwhelmingly on foreigners both for mathematically skilled positions in industry and for graduate students in pure math. I did once serve as a "de facto mentor" for a woman's undergraduate thesis. She did a super job, went on to graduate school, and is now a tenured math professor. I don't really see this as a "diversity issue," though. She did the same project a male student would have done. So in that sense mathematics is pretty gender-neutral. It's the social environment where things get messy, but that seems to vary enormously, from departments where gender isn't an issue to other departments that openly discourage women students and faculty.

Did you have support from family and society or face any barriers?

I don't think my parents ever grasped why I wanted to go into math (even when I was applying to graduate school my dad sat down with me and earnestly and kindly suggested that I might want to consider business school as an alternative). On the other hand, my parents fully supported my decisions and encouraged me at every step of the way. Society too has been supportive. However, in graduate school, the pressure took all the joy out of it, and under those circumstances it was hard to make progress. I didn't start enjoying mathematical research until after I got my PhD. I've taken a non-standard path, but there's always been a way to make things work. [After several years of teaching undergraduate mathematics, he resigned to care for his newborn son, and continued his research as a freelance mathematician.] This is a great time to be a mathematician.

Describe the process of doing mathematics?

Everything I do I see as pictures. The mental images are totally convincing. In cosmic topology, the problems arise from anomalies in our observations of the universe. I like to start with the absolutely simplest case that isn't totally trivial. If it's too messy I'll spend a few days trying to understand more deeply what's going on, in hopes of finding a simpler proof or a more enlightening point of view. This is particularly true of calculations—a calculation can prove that something is true without telling you *why* it's true.

How do you get the flashes of insight that you need to do research?

If I could tell you that, I'd be a far more productive mathematician :-) Seriously, for me the key is to be well-rested, unrushed and undistracted. Math is discovered, not invented. For me there is no doubt. As you explore, you work your way through a lot of "false understandings" where things don't quite come together. Then, all of a sudden, things start falling into place. That's the moment you realize you're onto something. It's very much an experience of discovering something that's already there. I find that a lot of progress takes place in my subconscious. That is, I can go to bed totally confused about a question and wake up with an idea. Similarly, I might be, say, out for a bike ride and find an idea just pops into my head, without my having been aware that I was even thinking about the problem. (Your students should understand, though, that they can't expect an idea to literally come from nowhere—they have to immerse themselves in the question first!)
Adapted from excerpts taken from:
Jeff Weeks *Exploring the Shape of Space*
my Jeff Weeks Interview

review

- Equations are prevalent inside and outside of mathematics and even though this geometry segment was not as equation heavy as the algebra segment, we still saw plenty of equations. List the instances that equations appeared in our geometry segment (words that describe the situation and/or equations themselves are fine). List as many instances as you can remember.

- Review these activities that we used to explore the geometry of the earth on the child's ball—what we did in the activities and what they showed us, and which activity did you find most convincing?

  a) car activity
  b) masking tape activity
  c) equator activity
  d) Chicago-Rome activity
  e) two great circles activity
  f) angle sum activity
  g) Pythagorean theorem activity

- When/how do higher dimensions exist in real-life data?

- What philosophical argument, experiment, or other justification is most compelling to you about whether the universe has finitely many or infinitely many stars?

- What philosophical argument, experiment, or other justification is most compelling to you about whether the universe has a specific shape or not?

- Choose a method or experiment that researchers have employed to determine whether our universe satisfies the laws of Euclidean, spherical, or hyperbolic geometry—what did they do and what did it show us?

- Discuss 2 of our classroom critiques of the method or experiment that you selected in the last question.

- List an instance from the geometry segment where the theme of local to global played a role. What was local? global?

- Critically analyze the role of probability and chance in the density experiments from the Jeff Weeks density experiments video: how do $\frac{1}{3000}$ and (separately) "plus or minus" $\pm$ (margin of error for the curvature confidence interval) relate?

- What real-life application was most compelling to you within our geometry class activities?

- Name a change in world view that came with mathematical discoveries from the geometry segment activities and the benefit that resulted.

- Reflect on this segment to discuss what is geometry?