What does a space look and behave like, how do we know, and how do we represent it?

1. In flat Euclidean geometry of the infinite blackboard from high school, named for Euclid of Alexandria (˜325 BCE–265 BCE), what is the sum of the angles in a triangle?

a) always 180 degrees

b) never 180 degrees

c) other

2. In M.C. Escher's (1898-1972) *Circle Limit 4: Heaven and Hell* representation of hyperbolic geometry, what is the sum of the angles we calculated from an angel's feet and 2 wingtips?

a) 180 degrees

b) 150 degrees

c) other

# *How do we Represent Hyperbolic Geometry?*

`http://www.beriewede.com/`

*Circle Limit 4: Heaven and Hell* by M.C. Escher, 1960

# *How do we Represent Hyperbolic Geometry?*

http://www.beriewede.com/

*Circle Limit 4: Heaven and Hell* by M.C. Escher, 1960

What are local to global connections?

–local to global artists and mathematicians contributing from Italy (perspective), Latvia/US (Daina Taimiņa), the Netherlands (MC Escher) and more today and in this segment

–geometry in local to global regions

https://www.geogebra.org/m/svywsx3r

3. If we had another work from Escher where there were 5 creatures around one point, 6 creatures around another, and 8 creatures around the third, respond to **all of the following**

a) What would the sum of the angles be?

b) Would the space be hyperbolic?

c) What would the angle sum tell you in terms of how curved the triangle is—is it very curvy?

4. In perspective drawing (projective geometry), parallels from the real world

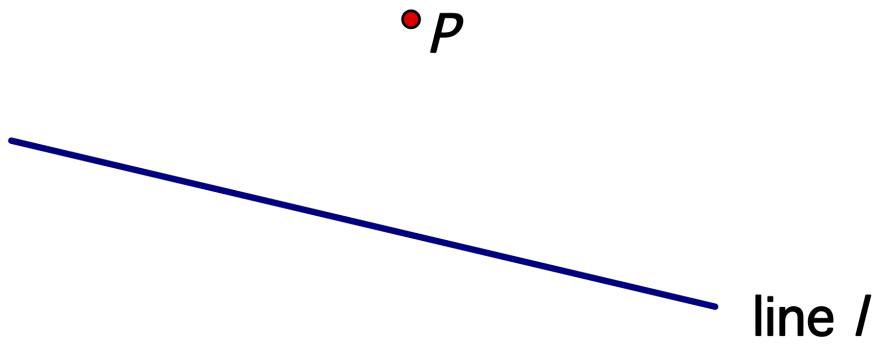
a) intersect in a vanishing point so there are zero parallels in this geometry, like spherical geometry
b) are drawn to intend to look parallel to the viewer
c) both

4. In perspective drawing (projective geometry), parallels from the real world
   a) intersect in a vanishing point so there are zero parallels in this geometry, like spherical geometry
   b) are drawn to intend to look parallel to the viewer
   c) both

5. In Euclidean geometry, John Playfair's (1748-1819) postulate that there is only one parallel to a line $l$ through a given point $P$
   a) always holds as we folded the perpendicular to the perpendicular, creating the unique parallel to $l$ through $P$
   b) never holds

4. In perspective drawing (projective geometry), parallels from the real world
   a) intersect in a vanishing point so there are zero parallels in this geometry, like spherical geometry
   b) are drawn to intend to look parallel to the viewer
   c) both
5. In Euclidean geometry, John Playfair's (1748-1819) postulate that there is only one parallel to a line $l$ through a given point $P$
   a) always holds as we folded the perpendicular to the perpendicular, creating the unique parallel to $l$ through $P$
   b) never holds
6. In Escher's hyperbolic space, John Playfair's (1748-1819) postulate that there is only one symmetric path cutting creatures (symmetry!) through a given point $P$ that never intersects another symmetric path $l$
   a) always holds
   b) never holds as we saw 3 such paths, not 1 unique path
   c) other

*Perspective is an effect brought about by distance.*



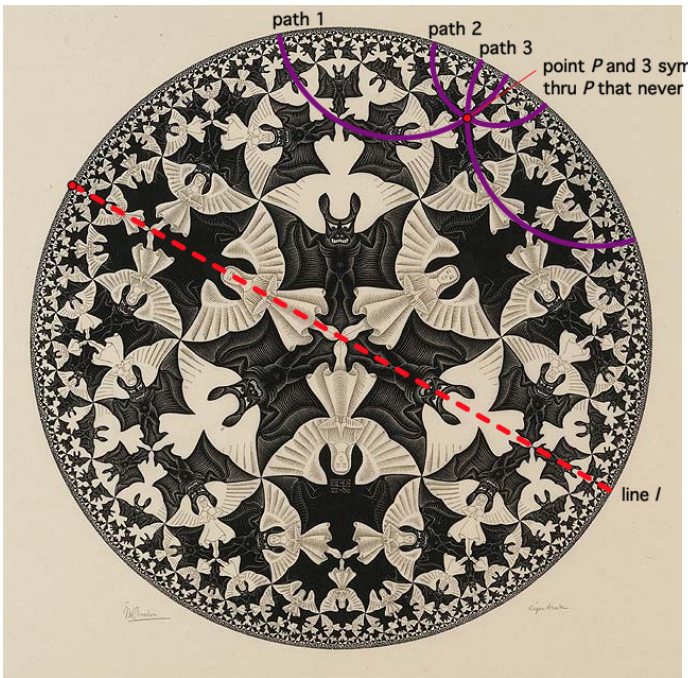
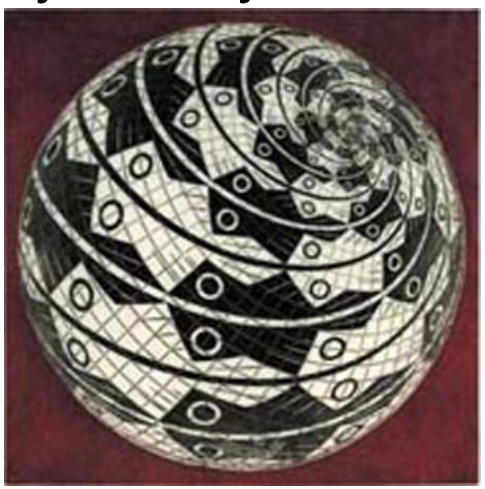
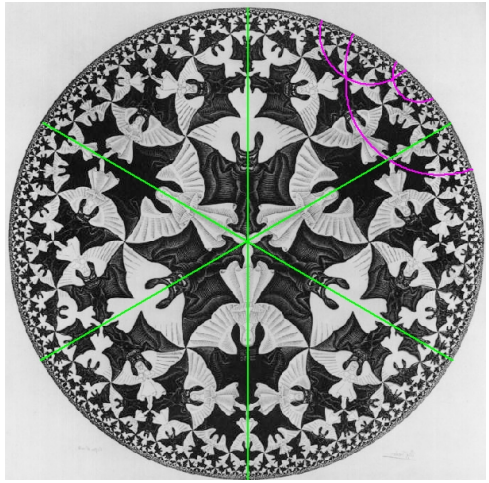




Image 1: `http://www.arthursclipart.org/sciencetechnology/`
Image 3: symmetric lines in Escher's *Circle Limit IV—Heaven and Hell*

# *What is Straight on a Curved Surface?*

symmetry!

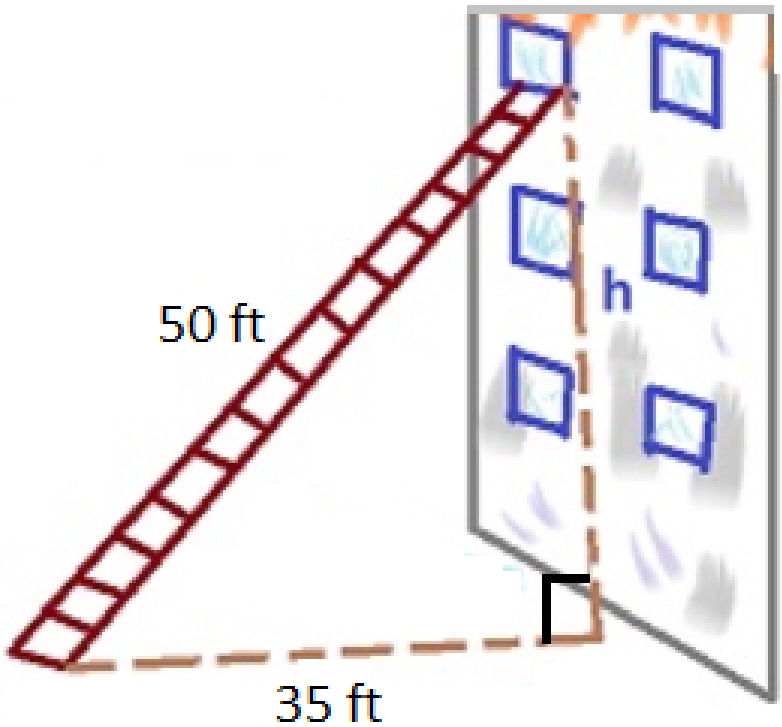




Dutch graphic artist M.C. Escher's Sphere Surface with Fish, 1958 and Circle Limit IV: Heaven and Hell, 1960;

Latvian/US mathematician Daina Taimina *Crocheting Adventures with Hyperbolic Planes*

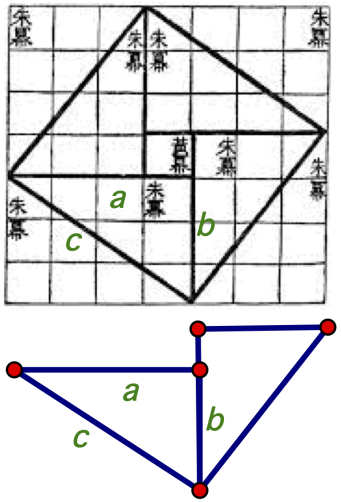
locally, how do we know if we are on a curved space or flat Euclidean space? angle sum or Pythagorean theorem

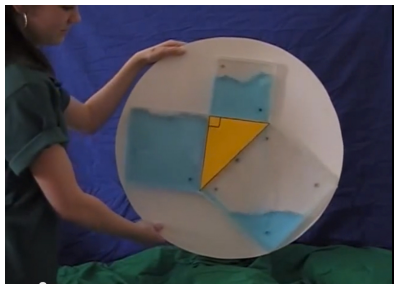
7. My husband is a professional musician who, in his spare time, volunteers for our local fire department and the rescue squad, as an EMT. If the firefighters have a 50 ft ladder and angle it towards the building, 35 ft away from the fire, how many feet high will it reach?

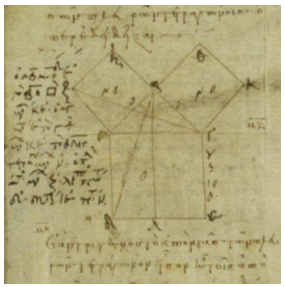


# Why is the Pythagorean Theorem True?

- Bhaskara's puzzle in *The Heart of Mathematics*



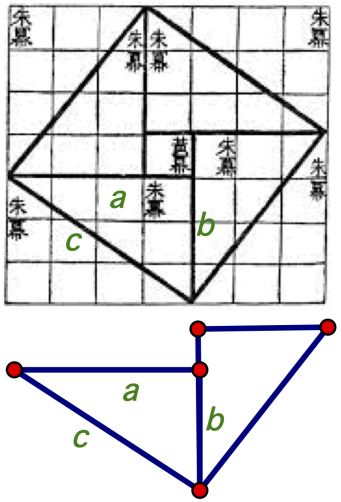


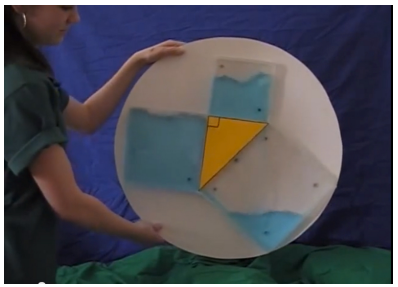
*Zhou Bi Suan Jing* or *Chou Pei Suan Ching*

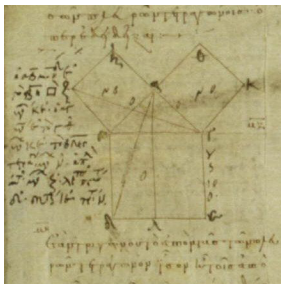
`https://www.youtube.com/watch?v=CAkMUdeB06o`, Euclid

# Why is the Pythagorean Theorem True?

- Bhaskara's puzzle in *The Heart of Mathematics*



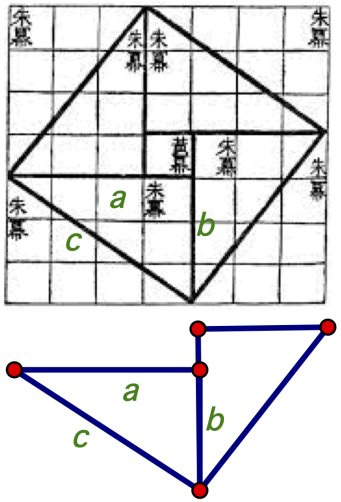


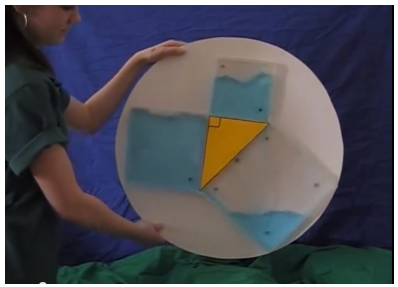
*Zhou Bi Suan Jing* or *Chou Pei Suan Ching* `https://www.youtube.com/watch?v=CAkMUdeB06o`, Euclid

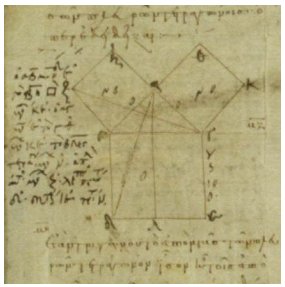
- large square has side $c$

# Why is the Pythagorean Theorem True?

- Bhaskara's puzzle in *The Heart of Mathematics*





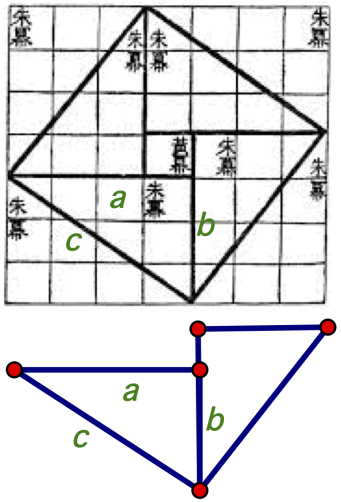
*Zhou Bi Suan Jing* or *Chou Pei Suan Ching* `https://www.youtube.com/watch?v=CAkMUdeB06o`, Euclid

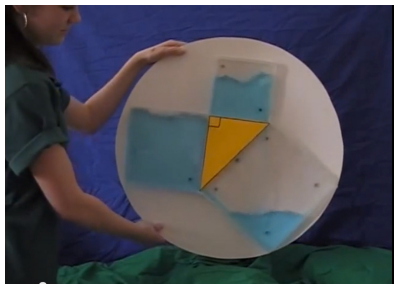
large square has side $c$

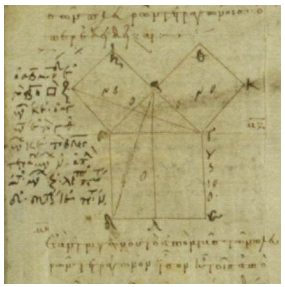
small square has side

# Why is the Pythagorean Theorem True?

- Bhaskara's puzzle in *The Heart of Mathematics*



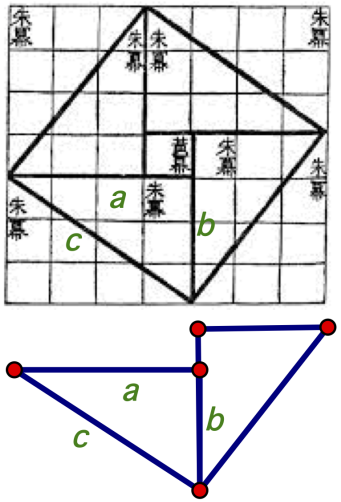


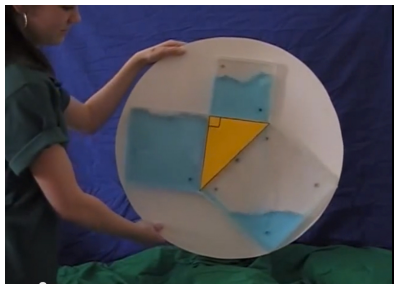
*Zhou Bi Suan Jing* or *Chou Pei Suan Ching* `https://www.youtube.com/watch?v=CAkMUdeB06o`, Euclid

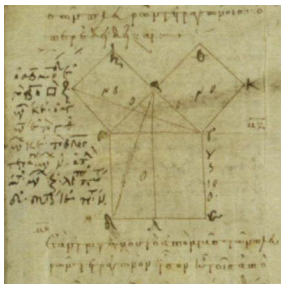
large square has side $c$
small square has side $a - b$

# Why is the Pythagorean Theorem True?

- Bhaskara's puzzle in *The Heart of Mathematics*



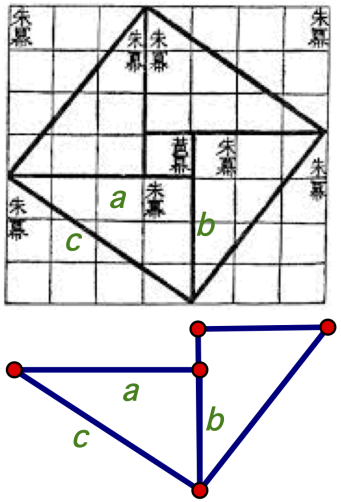


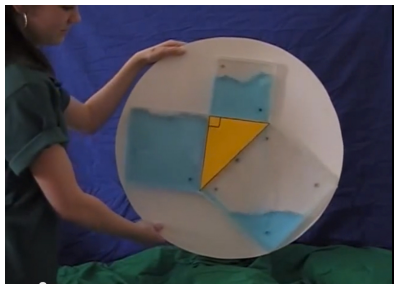
*Zhou Bi Suan Jing* or *Chou Pei Suan Ching* `https://www.youtube.com/watch?v=CAkMUdeB06o`, Euclid

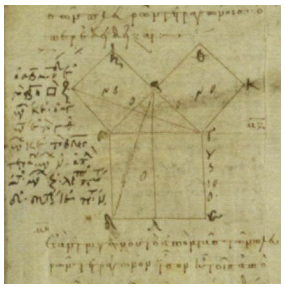
large square has side $c$
small square has side $a - b$
area of the large square = small square + 4 triangles
$c^2 =$

# Why is the Pythagorean Theorem True?

- Bhaskara's puzzle in *The Heart of Mathematics*



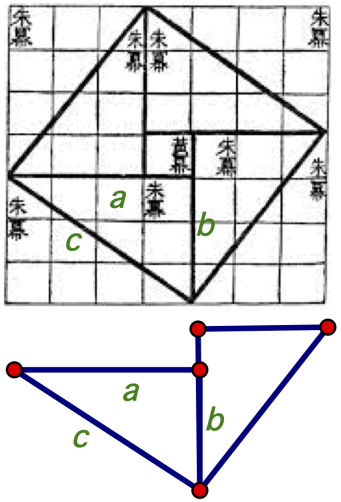


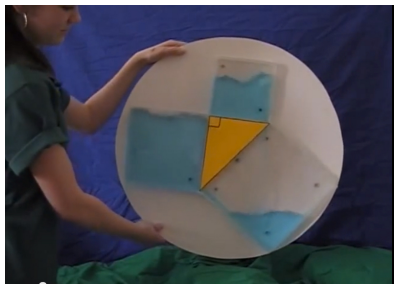
*Zhou Bi Suan Jing* or *Chou Pei Suan Ching* `https://www.youtube.com/watch?v=CAkMUdeB06o`, Euclid

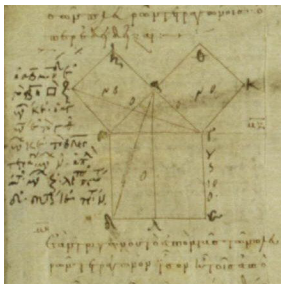
large square has side $c$
small square has side $a - b$
area of the large square = small square + 4 triangles
$c^2 = (a-b)^2 + 4(\frac{ab}{2})$

# Why is the Pythagorean Theorem True?

- Bhaskara's puzzle in *The Heart of Mathematics*

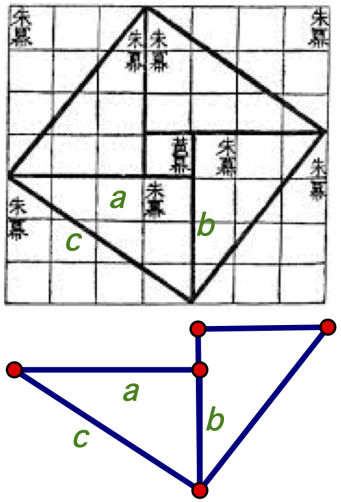


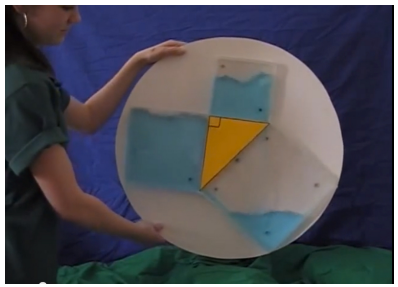
*Zhou Bi Suan Jing* or *Chou Pei Suan Ching* `https://www.youtube.com/watch?v=CAkMUdeB06o`, Euclid

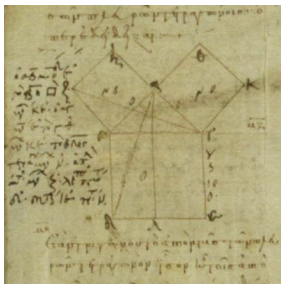
large square has side $c$
small square has side $a - b$
area of the large square = small square + 4 triangles
$c^2 = (a-b)^2 + 4(\frac{ab}{2})$
$c^2 = a^2 - 2ab + b^2 + 2ab$

# Why is the Pythagorean Theorem True?

- Bhaskara's puzzle in *The Heart of Mathematics*



*Zhou Bi Suan Jing* or *Chou Pei Suan Ching* `https://www.youtube.com/watch?v=CAkMUdeB06o`, Euclid

large square has side $c$
small square has side $a - b$
area of the large square = small square + 4 triangles

$$c^2 = (a-b)^2 + 4(\frac{ab}{2})$$
$$c^2 = a^2 - 2ab + b^2 + 2ab$$
$$c^2 = a^2 + b^2$$

# Pythagorean theorem in Hyperbolic Geometry?

https://www.geogebra.org/m/jxejcdy5

# Where is the Pythagorean Theorem Useful?

Babylonian cuneiform Plimpton 322

cuneiform

# Where is the Pythagorean Theorem Useful?

Babylonian cuneiform Plimpton 322

cuneiform

Baudhayana sutras (Vedic Sanskrit texts) also predate Pythagoras.

दीर्घचतुरश्रस्याक्षणया रज्जु: पार्श्वमानी तिर्यग् मानी च यत् पृथग् भूते कुरुतस्तदुभयं करोति ॥

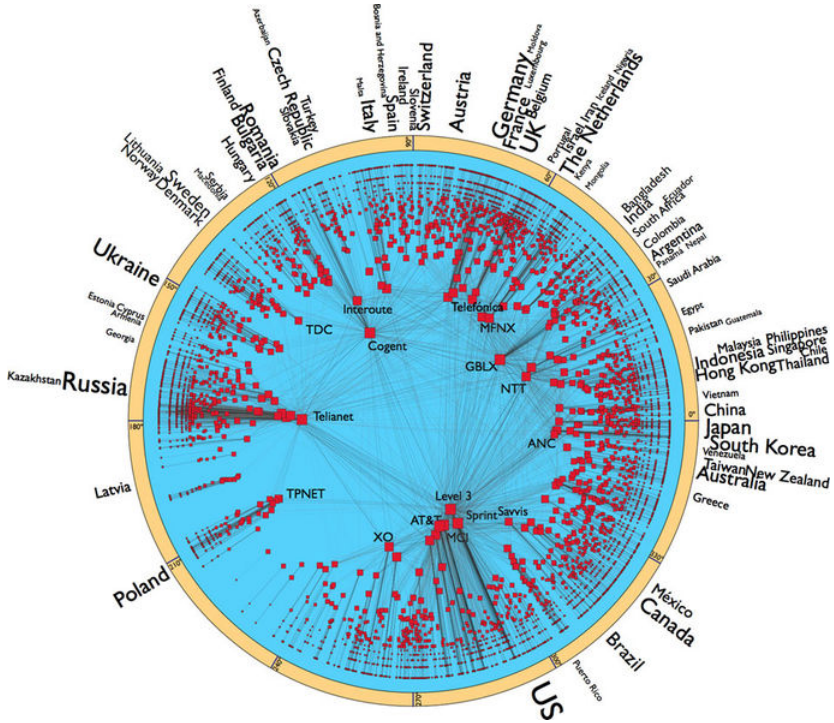
A rope stretched along the length of the diagonal produces an area which the vertical and horizontal sides make together.

Egypt 3, 4, 5 right triangles

astronomy, engineering, construction

# *non-Euclidean 2D Geometries and their Applications*

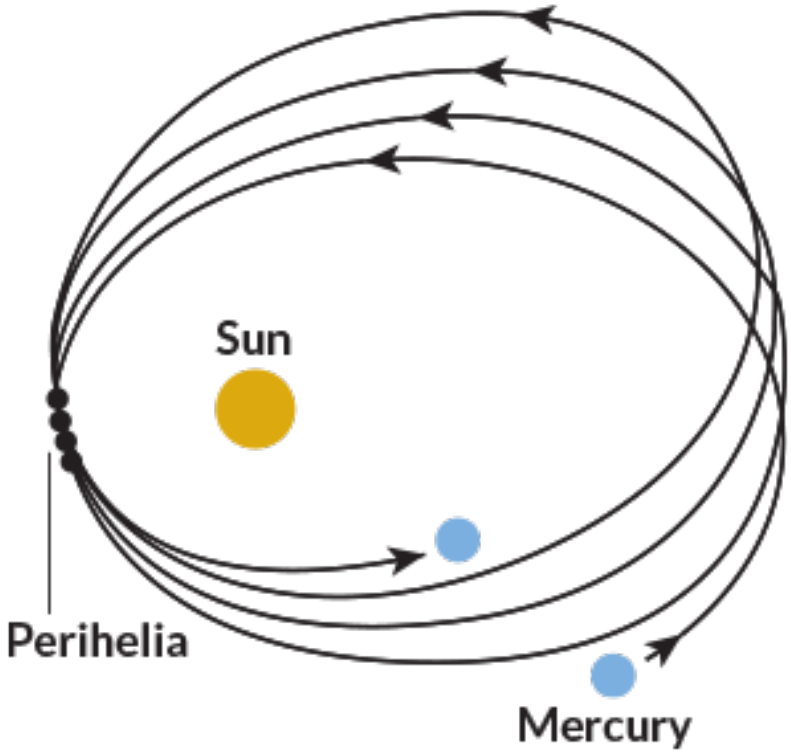
- Models of the internet to reduce the load on routers




Sustaining the Internet with hyperbolic mapping: Marian Boguna, Fragkiskos Papadopoulos & Dmitri Krioukov

- Building crystal structures to store more hydrogen or absorb more toxic metals

# *Modeling and Explaining Real-Life Behavior*




www.sciencenews.org/article/einsteins-genius-changed-sciences-perception-gravity

- hyperbolic geometry better models Mercury's orbit
- both Euclidean and non-Euclidean geometry map the brain to diagnose or monitor neurological diseases

# *Modeling and Explaining Real-Life Behavior*

life.dpics.org

CC-BY-2.0 Margaret Wertheim

Crochet Coral Reef: an ever-evolving nature-culture hybrid

`https://crochetcoralreef.org/`