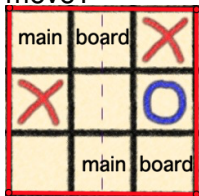


Clicker Question

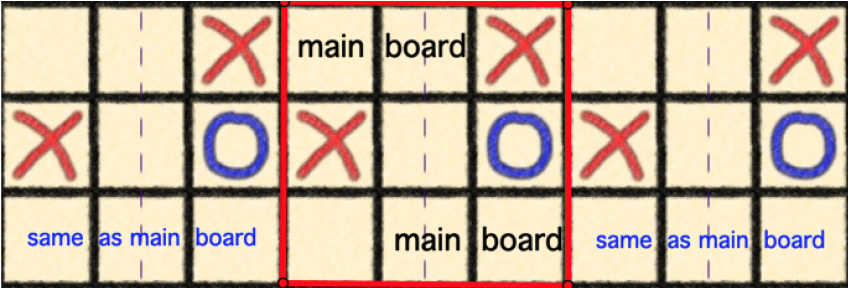
1. In this Klein bottle universe, named for Felix Klein, first sketch a tiling view.

Next, Where should o go to prevent x from winning on the next move?

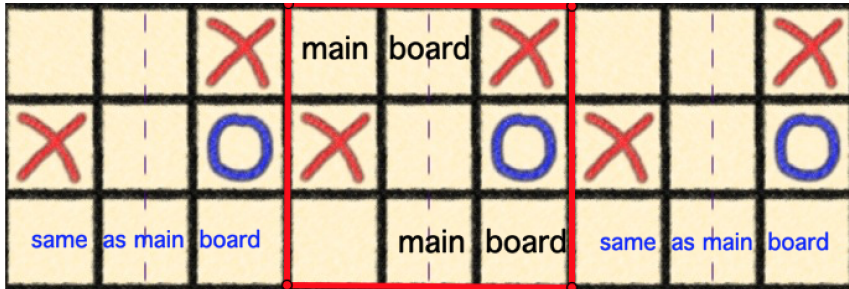


- a) top left square
- b) Euclidean geometry
- c) middle square
- d) bottom left square
- e) x has no chance of winning so o can go anywhere
- f) x can win no matter where o goes

Klein bottle tic-tac-toe: Where should \circ go next on the main board?

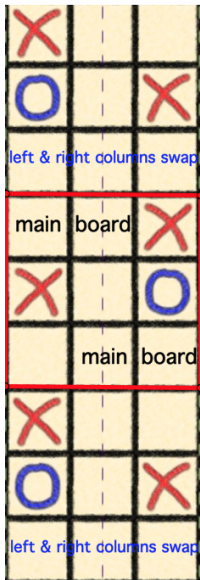


Klein bottle tic-tac-toe: Where should \circ go next on the main board?



bottom middle to block \times

Klein bottle tic-tac-toe: Where should **o** go next on the main board?



also bottom left to block **x**, so **o** has no chance of winning



Clicker Question

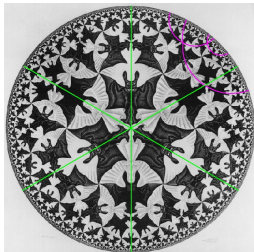
2. In 4.6: The Shape of Reality? a saddle was an example of what kind of geometry?

- a) Euclidean (flat)
- b) spherical
- c) hyperbolic
- d) more than one of the above
- e) none of the above

Clicker Question

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Latvian/US mathematician Daina Taimina *Crocheting
Adventures with Hyperbolic Planes*

Clicker Question

3. Does Carl Friedrich Gauss's mountain peak experiment to measure the angle sum prove that the universe is flat?

- a) Yes and I have a good reason why
- b) Yes but I am unsure of why
- c) No but I am unsure of why not
- d) No and I have a good reason why not



Clicker Question

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result was within $1/180$ th of 180 degrees.

Experimental error provides an estimate of the inherent uncertainty associated with experimental procedures. This is quantified in many experiments as a margin of error.

Clicker Question

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Critiques: Experimental error, light rays bend with gravity, triangles too small, convenience sample

Clicker Question

4. Which did you find most compelling for why great circles are intrinsically straight and shortest distance paths?

- a) string pulled tightly
- b) masking tape
- c) toy car
- d) symmetry
- e) other

Clicker Question

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5. Which were valid arguments?

- a) If the definition of parallel is intrinsically straight paths that never meet, then there are no parallels on the sphere
- b) If the definition of parallel is paths that never meet, then there are parallels on the sphere
- c) both of the above

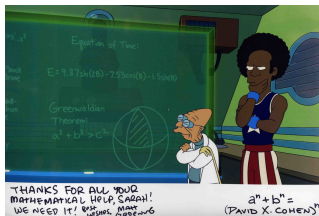
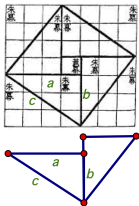
Clicker Question

6. Why is the Pythagorean theorem false on the sphere?
- a) *Futurama* says so
 - b) Because we can create the two base sides a and b with string, flatten and create c_{flat} , and put it back on the sphere to see that it is too long, ie: $a^2 + b^2 = c_{flat}^2 > c_{sphere}^2$
 - c) both of the above
 - d) none of the above, because the Pythagorean theorem holds on the sphere

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勾股定理何以成疑



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- 5) **Problem 5: Is our universe 3-dimensional or is it higher dimensional? Why?**
- 6) Problem 6: Are there are finitely or infinitely many stars in the universe? Explain.
- 7) Problem 7: We know that the shape of the earth is close to a round sphere. Could the universe be round too? Does it have any kind of shape?

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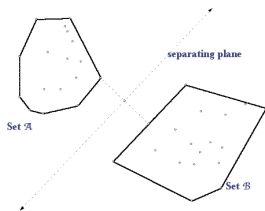
7. Where does *The Heart of Mathematics* address higher dimensional spaces?

- a) artistic representations
- b) mathematical spaces
- c) real-life data
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Clicker Question

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The Shape of Space



Hubble: Space Telescope Science Institute

Experiment: Looking for repeated star patterns

Critiques: light takes times to reach us and changes the view, recognize?

What sequence (over time) would we see if a hypersphere passed by us? Think about an analogue of a sphere passing by a 2-D creature's plane of existence, like when Spherius passed by Arthur Square in *Flatland*.

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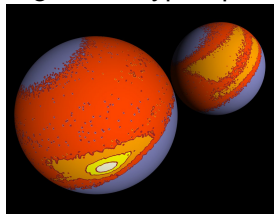
<https://vimeo.com/73243719>

Algebraic hypersphere: $x^2 + y^2 + z^2 + w^2 = 1$

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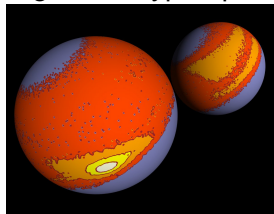
The overall shape of a protein can be parameterized as a sequence of points on the unit sphere. Shown are two views of the spherical histogram of such points for a large collection of protein structures. The statistical treatment of such data is in the realm of directional statistics. [Thomas Hamelryck]

Applications: AI, biology, machine learning, statistics, and Einstein's theory of relativity.

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<https://www.youtube.com/watch?v=kNva9WpQXvM>

WMAP and Planck satellite data for density suggest an infinite Euclidean universe within .4%

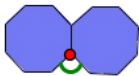
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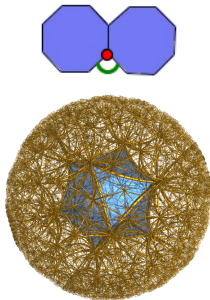
- truth may not be obtainable with too small a sample
- look for reproducibility and consensus
- Understanding what the possible underlying variables are (do supernovas explode at the same brightness?) and conducting sufficiently controlled experiments with alternative hypotheses (can we explain the missing fluctuations in WMAP data?) are the key to getting at truth.
- We get to truth when we have enough diverse evidence all showing the same result (the earth is not flat). The universe may satisfy the laws of Euclidean geometry (flat)

<https://www.youtube.com/watch?v=ujYcn00sIks>

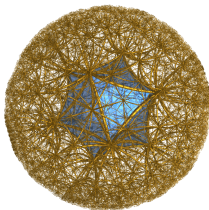
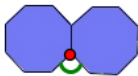
<https://www.youtube.com/watch?v=jFT1pxHmSWA>



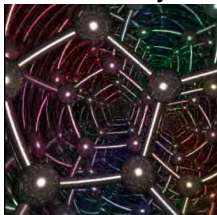




Hyperbolic Icosahedron by Claudio Rocchini



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Life from the inside [spherical dodecahedron] from Paul Nylander