Geometry Intro hand in
Dr. Sarah's MAT 1010: Introduction to Mathematics
Geometry of the Earth and Universe
How we measure and view the world around us and decide what is the nature of reality.

submission instructions: Must be completed on this handout and collated into one single PDF for submission in the geom intro assignment

goals:

- Explore applications of geometry in everyday life.
- Identify geometric ideas/connections within local and global geographical regions.
- Recognize patterns and similarities in algebraic representations and employ spreadsheets and graphing software to apply those representations to the real-world. Utilize technology to adapt and use mathematical formulas that include cell referencing to answer real-world questions and interpret results.
- Communicate geometric information in written documents.

I'll ask you about the **people and their contributions**, so keep track of those in this handout.

Shape of the World & *Seeing is Believing* from *Life by the Numbers* Video

Access and watch the ***Life by the Numbers*** video from ASULearn. Be sure you are logged in to your appstate Google account. If you are logged in to numerous Google accounts, you may need to log out of any others. The people in the video:

American Actor & Director (narrator in this video): **Danny Glover**
British Artist and Mapmaker: **Nigel Holmes**
American Art Historian: **Sam Edgerton**

The video description: "Mathematics. We'd all be lost without it. Literally. Without mathematics we'd have no maps, no globes, no idea what the world really looks like. From Renaissance paintings to major motion pictures, the world of mathematics plays a pivotal role."

1. What are the ways that the people talk about, explore or represent the earth throughout the video (what do they say about it, what are the analogies, visual representations...)?

   List as many as possible.

2. What does geometry mean from the video?

3. What are the challenges with maps and geometric representations in the video?

4. What are the advantages of perspective geometry in the video?

5. Identify an instance from the video of geometric ideas/connections within local (small-scale) and global (large-scale) geographical regions. List them as follows:

   local (small-scale) region: geometric ideas/connections:

   global (large-scale) region: geometric ideas/connections:

Note that for the upcoming think-pair-share assignment you'll need to summarize in your own words how Sam Edgerton discusses the connection to the industrial revolution, so you may want to make note of this now.

Parallels and Perspective

One definition of parallels that will extend well to numerous spaces is straight-feeling paths that never meet. In Euclidean geometry, the flat geometry from high school (named for **Euclid of Alexandria**), there is exactly **one parallel** to a line through a given point. In fact we can show why and find it by folding as follows.

6. From the ASULearn assignment, access **foldingparallel.pdf** and print it or trace it to make a physical copy. Next, make a fold through point P so that the line overlaps itself and creates a perpendicular to line l through point P so that you turn [diagram: point P, line l] into [diagram: point P, line l, fold so line l overlaps itself] .
Using a pen or marker, draw the perpendicular line you folded—on both sides of the paper—and go past P as in [diagram: P, line l] . Next fold a perpendicular to the line you just drew through the point P. To do this, make a fold through point P so that the perpendicular line you just drew overlaps itself as in [diagram: P, fold so the perpendicular overlaps itself, line l] . You should have created the **unique parallel** to line l through point P. Does it look parallel (If not, try again)?
Circle one: yes, on the first try   yes, eventually   no/unsure—I'll try for a Zoom

7. Place your completed folding **on or near this page to scan in** when you convert this document to a single PDF. For example, it could fit on top of text at the start of this Parallels section.

This folding activity is similar to Euclid's original construction of a parallel in Proposition 31 of Book 1 of Euclid's *Elements*. Carl Boyer, a historian of science, called Euclid's *Elements* the most fabulously successful and influential textbook of all time because of its reach and importance in the development of mathematics and science.

In non-Euclidean spaces, straight to us means that our feet will travel on symmetric paths, i.e. that creatures will get cut in half as they travel along such a path.

8. Here on the left is Dutch graphic artist **M.C. Escher** working on one of his **sphere surfaces** with fish. He drew the symmetric path through each fish, cutting it in half, showing they are swimming along straight-feeling paths. All of these meet up at one point, like a North pole. Label this point. Since all straight-feeling paths intersect, there are **no parallels** in this particular work of Escher's.

Label the intersection point below and

draw a line from it

Escher working on sphere surface with fish
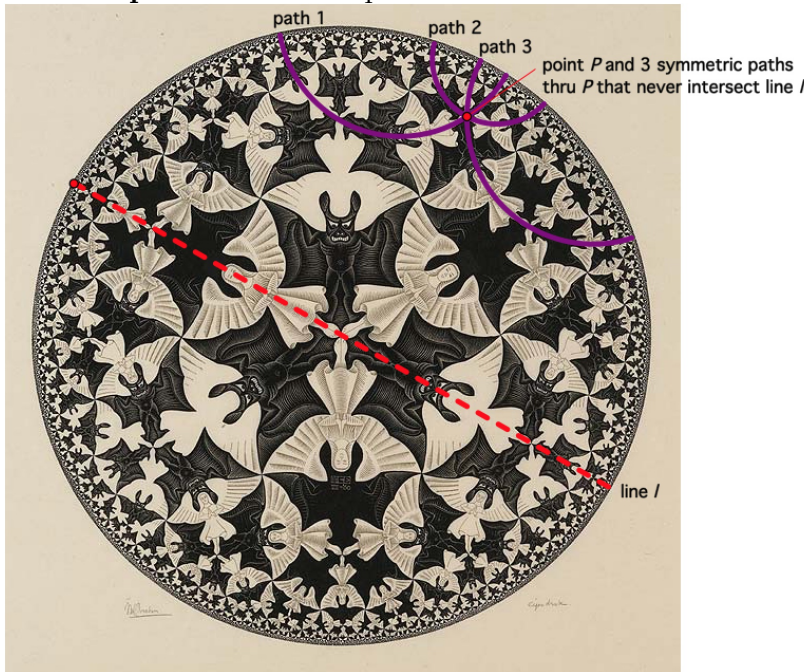
symmetric parallels to line *l* through point *P* in hyperbolic surface

9. Above on the right, **hyperbolic geometry**, another non-Euclidean geometry, is different than spherical geometry. Count how many parallel paths are through point *P* to line *l* in Escher's *Circle Limit IV—Heaven and Hell* representation from 1960 (use the annotations I added to help you)?

10. As in the video, in **perspective drawing**, lines that were parallel in the real-world intersect in a vanishing point. Using the definition of straight-feeling paths that never meet, are there any parallels in this geometry? **Circle one**: yes   no   unsure—I'll try for a Zoom

11. In terms of the number of parallels, which geometry is perspective drawing most similar to?
Circle one: flat Euclidean geometry (1)   spherical geometry (0)   hyperbolic geometry (more)

Come One—Come All—to a Better Cube

12. Here we will apply the following to the image below, a very simplified version of British artist **Julian Beever's** pavement drawings we looked at.

- Use a thumb and a forefinger to measure the viewing distance *d* between *V* and the second dot in the picture below. Keep this distance, called *d*, with your hand.

- Continuing to keep that distance, turn your hand to measure out d units directly in front of V so that d is now perpendicular to the page/coming out of the page instead of being inside of it.
- Place your **left eye** exactly at that same point (d units out in front of V). You will need to get **very close** to the handout to do this.
- Close your **right eye**.
- Without changing your position, let your left eye roll down and to the left and then look at the box. Although it may be too close for comfortable viewing because you will be so close, the distortion should go away and it should look much more like a cube! Most people will be able to see this by following the directions, but some people may have problems due to astigmatism...

viewing distance d

V
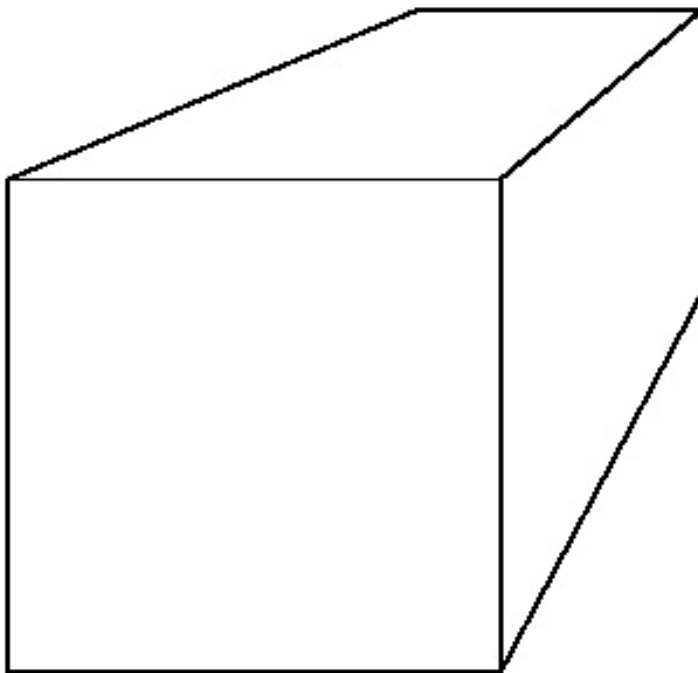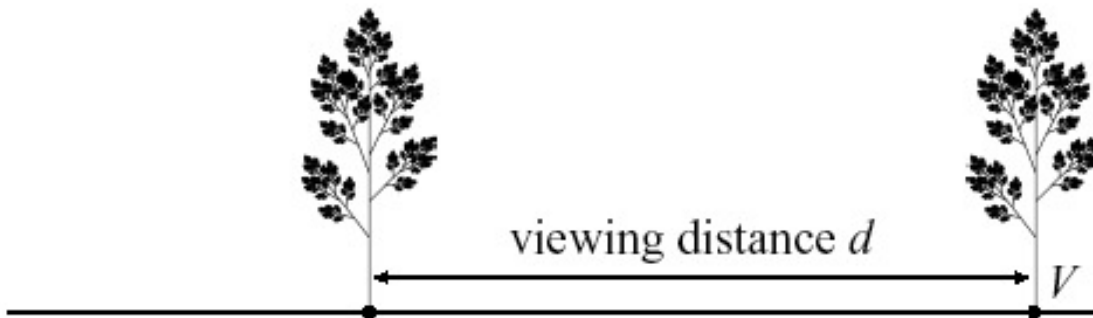
Circle one: Were you able to see the distortion mostly disappear when your left eye was d units out of the page in front of V?   yes   somewhat   no   unsure—I'll try for a Zoom

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Using Mathematics to Create Precise Perspective Drawings

Experiments with perspective drawing were completed long ago, when people with interdisciplinary interests (like math and art) were perhaps even more common than today. In this woodcut by German artist and theorist **Albrecht Dürer**, the screw eye on the wall is the desired viewer's eye, the lute on the left is the object, the taught string is a light ray, and the picture plane is mounted on a swivel.
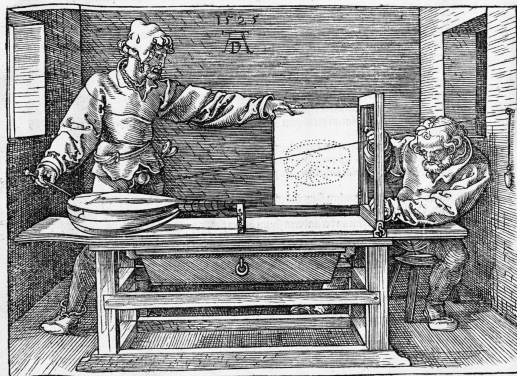
"Unterweisung der Messung" by Albrecht Dürer, 1525

Leonardo Da Vinci and **Brook Taylor** researched the question of how to find the viewing distance of a painting, and Taylor's 1715 work was published in *Linear Perspective: Or, a New Method of Representing Justly All Manner of Objects as They Appear to the Eye in All Situations*

These mathematicians and artists found the precise mathematical rules for perspective drawing. Understanding just a little bit about these rules can help us understand art and computer animation.

13. Download and open this Excel file **perspectivehouse.xls** from this ASULearn assignment. You will see a chart that is partly filled in with real-life coordinates x, y, z of a house (in columns A, B and C, respectively). We will use the viewing distance of 15 (as in column D) to calculate x' and y', and create a perspective drawing of it in Excel. So, we want to mathematically project the three-dimensional house onto the mathematically precise perspective image in the plane. So, we want to transform x, y and z to new coordinates $x' = \dfrac{dx}{z+d}$ $y' = \dfrac{dy}{z+d}$. We will make Excel do these formulas for us!

    To transform x to x', click on $E2$ in the Excel sheet (row 2, column E). The Excel version of the formula for x' that you should type in is:

    =D2*A2/(C2+D2)

    so type this formula (including the equals sign) into $E2$ and hit return. You should now see -1.875. Since d is in the $D2$ box, x is in the $A2$ box, and z is in the $C2$ box, we see that the Excel formula that we just typed in is the correct formula to use in order to find the 2D perspective x-value of the given 3D point, since it corresponds (in the second row) to the formula $x' = \dfrac{dx}{z+d}$. Make sure you understand this before you continue and apply it.

14. To **fill down the formula to the other rows**, we'll force Excel to automatically change the 2s to 3s to 4s and so on. Click on $E2$ again. Move your cursor to the lower-right corner so that it turns into a black plus sign ![black plus sign] . Click so that you keep the black + sign, hold down, and fill down the Excel formula by scrolling down the column vertically and releasing in $E18$. The number you will see there is -2.7631579. If you have problems, watch the fill down helper video in the assignment link on ASULearn.

15. What Excel formula should we use to correspond to $y' = \dfrac{dy}{z+d}$ for the second row (don't forget the equal sign)? (Hint: it is very close to the Excel formula for $E2$ with one modification, so go back to #13 to look at x' and how it was written in Excel and change the one part that is different)

16. Enter your formula into $F2$ and hit return. You should now see -2.8125.

17. At the bottom corner of $F2$ scroll until you have the + sign, click, and then then fill down the Excel column and release in $F18$. The number you will see there is .39473684.

18. To draw our house, click on the grey E box, at the top of the E column above $E1$ so that that the entire column is highlighted.

19. Then hold down the command key while you click on the grey F box, so that both the E and F columns are now highlighted.

20. Excel has two types of menus, those at the very top of the screen, in grey background below, and those on the spreadsheet itself, which has a green background as below. Under the Insert top menu, release on Chart and then Scatter or XY (Scatter), as shown here, or use a different way to select this or something similar, depending on your version of Excel.
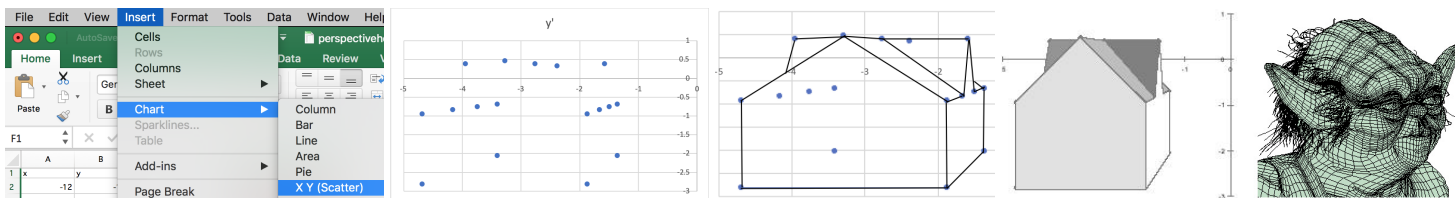
Ta-da, we have our mathematical drawing, but it doesn't look as impressive. Some of the points of the house are behind our viewpoint, and are only there if we want to change our view (like by making the house rotate). To have the computer complete the house drawing, all we would have to do is connect the dots and shade in the figure, as in the image above (but don't do this now). If you are having problems creating the graph in Excel, try again—you might have something extra selected instead of only the 2 columns.

21. Were you able to create it (if not, try again)?
Circle one: yes   no/unsure—I'll try for a Zoom

Digital animations such as use many more rows of Excel. The full-body version of this Yoda uses 53,756 points! Models were created by **Marc Frantz** for the cube and Excel house and **Kecskemeti B. Zoltan** for Yoda, who is visualized by **Tim Chartier**. Yoda courtesy of Lucasfilm LTD as on Using the Force of Math in *Star Wars*.

22. List the artists and mathematicians who are mentioned in this handout and give a very brief summary of their contributions. Example: Julian Beever, pavement drawings in perspective