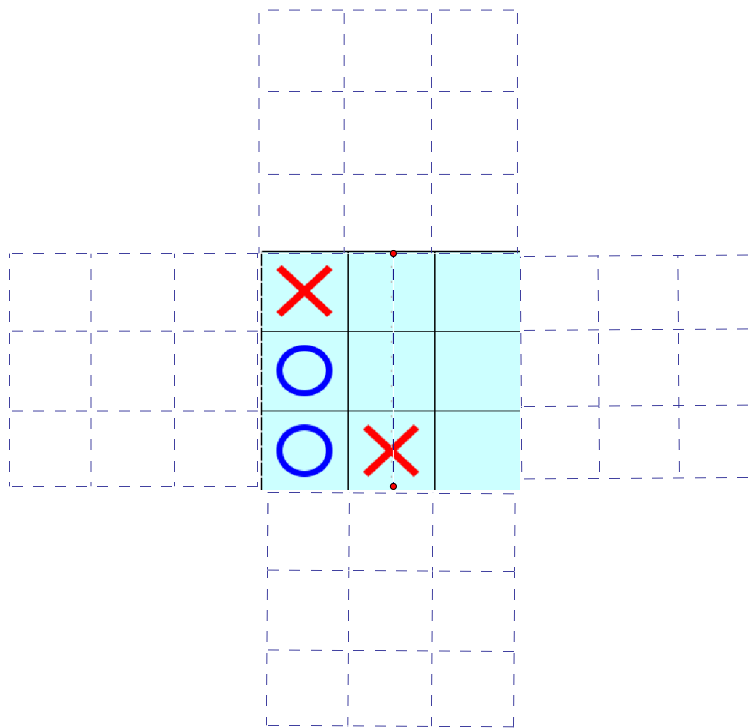
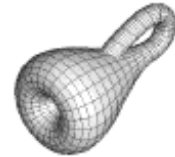




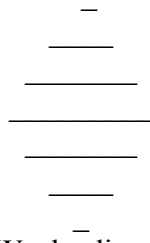
Jeff Weeks, 1956 - Present

Recall that one possible Euclidean 2-D Universe is a Klein bottle. In Klein bottle tic-tac-toe, a game that Jeff Weeks designed in order to share elements from his research with his son and others, we identify the left and right sides of the board straight across. The top and bottom are glued via a **reflection** in the vertical line through the middle. This would then form the Klein bottle you see. I am “X” in the game and I can win Klein bottle tic-tac-toe with my next move. Mark off where I should go to win. **Question 1:** Mark your answer on the board below and also draw a tiling view of the identified game pieces above, below, to the right and left of the game board.



We'll use some of Jeff Weeks' work along with a bit of *Futurama* to help visualize a finite universe without any edges that is obtained by identifying the faces of a cube.

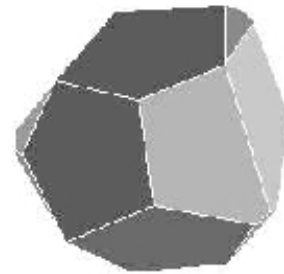
Recall that a 2-D Marge might see something like the following sequence (over time) if an orange passed by her plane of existence:



In the interview you read for homework, Jeff Weeks discusses his work on the hypersphere, the higher dimensional analogue of the sphere. In 1917, Albert Einstein used Bernhard Riemann's work on the hypersphere in order to present a model for the universe that was consistent with his theory of relativity.

Question 2: What sequence (over time) would we see if a hypersphere passed by us? Think about an analogue of a sphere passing by a 2-D creature's plane of existence. Hint: A sequence of spheres - how do they change?

Jeff Weeks has a hyperbolic universe named after him - the *Weeks manifold*, which he was the first to discover. To form the space, we identify similar faces like the two pentagonal faces, so that there are no edges to the space. In the same way we can identify opposite faces of a cube to obtain a Euclidean universe, and opposite faces of the dodecahedron to obtain a spherical universe. Mathematicians conjecture that this is the smallest such hyperbolic object.



Question 3: Search the web for information about hyperbolic web browsers, an application of hyperbolic geometry, or some other application of hyperbolic geometry.

References

- 1) Interview with Jeff Weeks (2005).
- 2) Allyn Jackson and Mary Beth Ruskai, *Maldacena, Shor, Silverstein, and Weeks Receive MacArthur Fellowships*, Notices of the American Mathematical Society, Vol 24, No. 8, 1999, 891-892.
- 3) Weeks, Jeffrey R., *Exploring the Shape of Space*, Key Curriculum Press, 2001, http://www.keypress.com/catalog/products/supplementals/Prod_ShapeOfSpace.html.
- 4) Weeks, Jeffrey R., *Exploring the Shape of Space Cosmology News*, Key Curriculum Press, <http://www.geometrygames.org/ESoS/CosmologyNews.html>.
- 5) Weeks, Jeffrey R., *The Shape of Space*, Marcel Dekker, 2002.