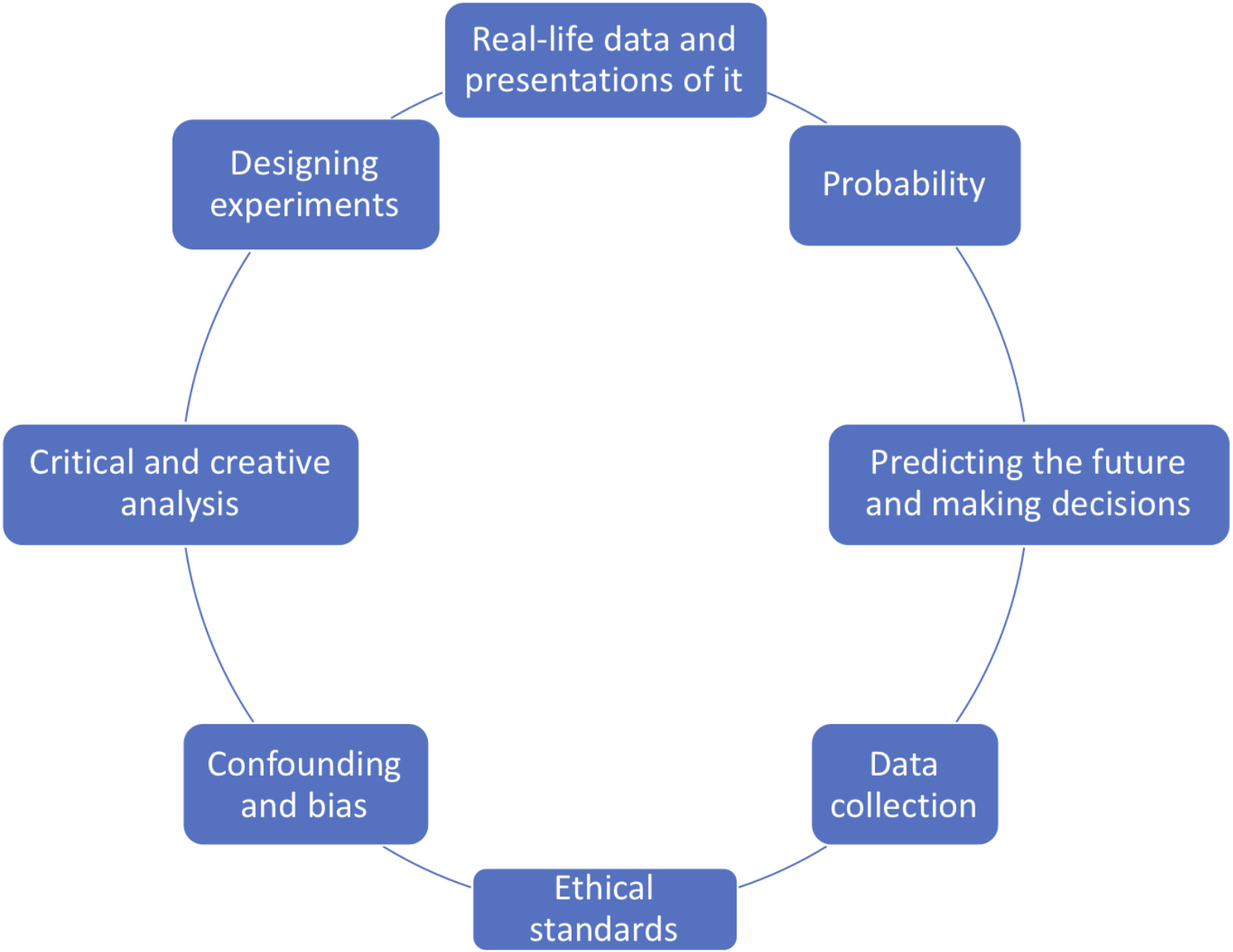
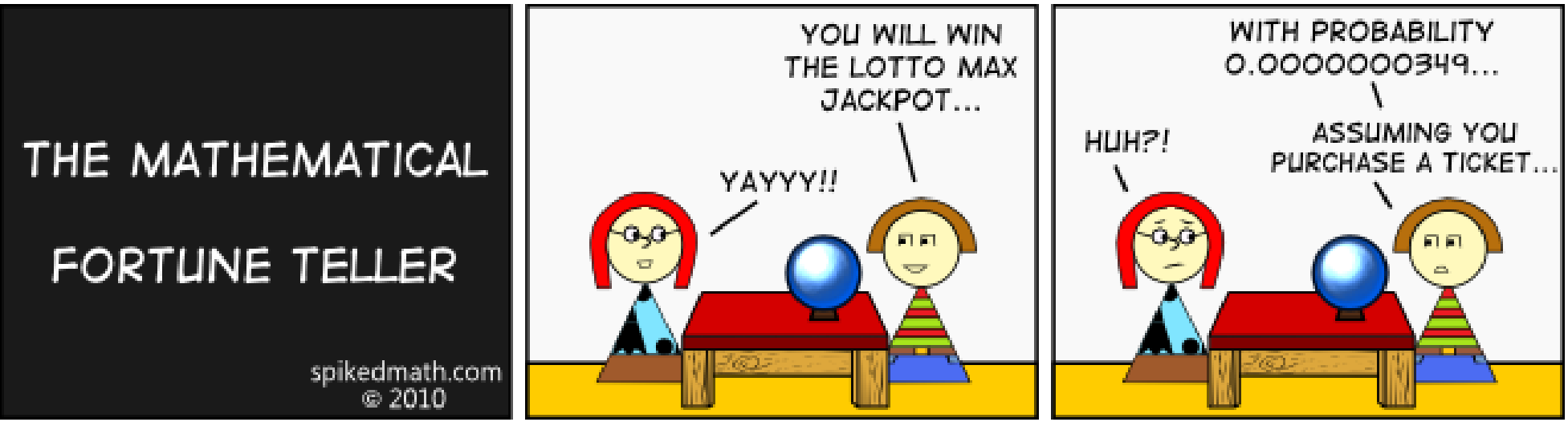
On the board, write the number of people who responded to your survey and whether any shared birthdays, like 34–yes



Coincidence and Uncertainty in Daily Life

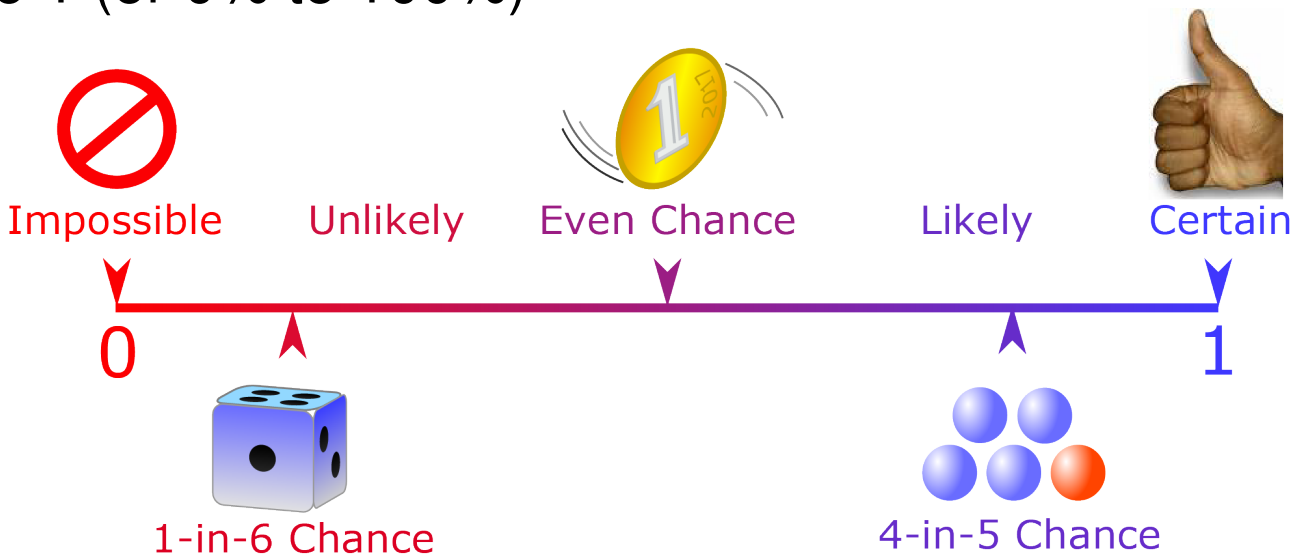
- many events in our daily lives arise in terms of probabilities and statistics—even the basic interactions of molecules and subatomic particles
- we can use probability to move beyond a vague sense of disordered randomness and describe possible outcomes



Picture credit: `http://spikedmath.com/355.html`

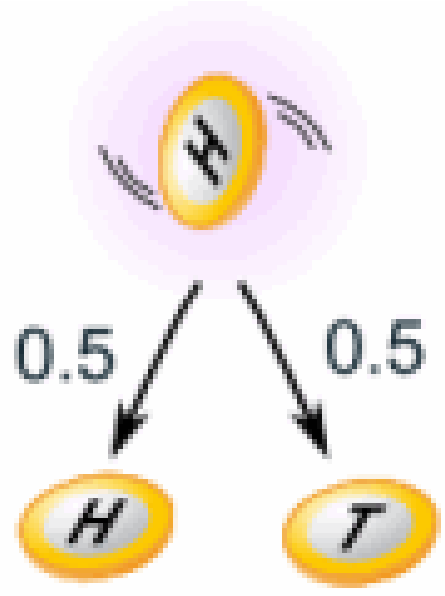
Probability

- quantitative measure of the likelihood of an event
- mathematical foundation of common sense and good judgment
- 0 to 1 (or 0% to 100%)



Picture credit: `https://www.mathsisfun.com/data/probability.html`

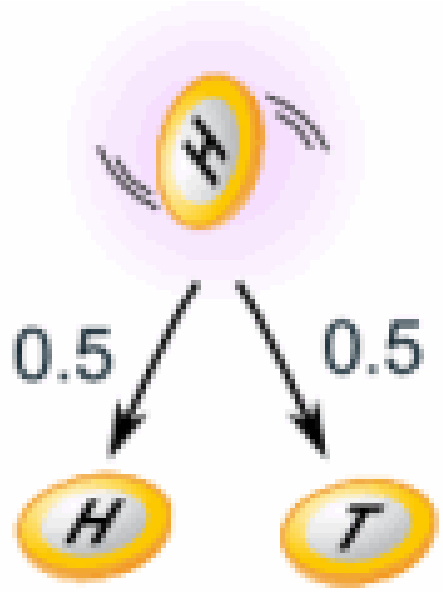
If It Either Happens or It Doesn't (Independent Events)



Picture credit: `http://lriser03.blogspot.com/`

- probability that an event will happen =
  1 - probability it won't happen
- What is the probability of NOT rolling a 6 on a dice?

If It Either Happens or It Doesn't (Independent Events)



Picture credit: `http://lriser03.blogspot.com/`

- probability that an event will happen = 1 - probability it won't happen
- What is the probability of NOT rolling a 6 on a dice?
  $1 - \frac{1}{6} = \frac{5}{6} = \frac{\text{number of different outcomes}}{\text{total number of equally likely outcomes}} =$ probability of rolling 1, 2, 3, 4 or 5.

Multiplication Rule for Independent Events

- If the probability of a person being left-handed is $\frac{1}{10}$, and the probability of being blue-eyed is $\frac{1}{3}$, then what is the probability of being left-handed and blue-eyed (assuming these are independent of each other)?

Multiplication Rule for Independent Events

- If the probability of a person being left-handed is $\frac{1}{10}$, and the probability of being blue-eyed is $\frac{1}{3}$, then what is the probability of being left-handed and blue-eyed (assuming these are independent of each other)?
- If independent, then the proportion of blue-eyed people among the left-handed people is the same as the proportion of blue-eyed people among the whole population, so
  left-handed and blue-eyed = $\frac{1}{3}$ of $\frac{1}{10}$ = $\frac{1}{3 \times 10} = \frac{1}{30}$

Happy Birthday to You and You!

- What is the probability of two people NOT sharing a birthday (month and day, and ignoring leap years)?

  A related question: Given person 1's birthday, how many other birthdays are there in a year that person 2 could have?

Happy Birthday to You and You!

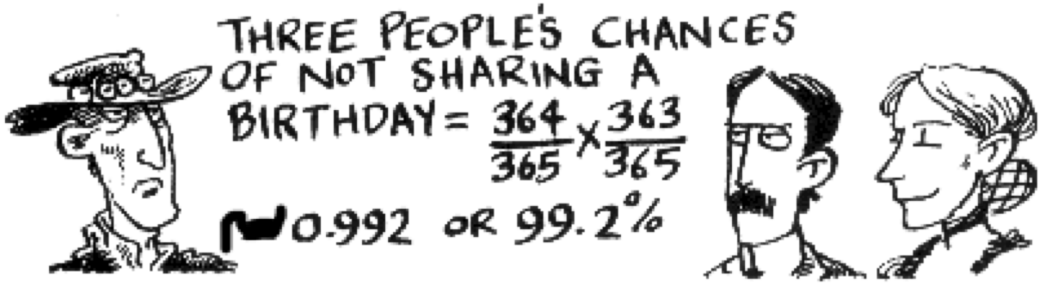
- What is the probability of two people NOT sharing a birthday (month and day, and ignoring leap years)?

  A related question: Given person 1's birthday, how many other birthdays are there in a year that person 2 could have?

Picture credit: `http://www.murderousmaths.co.uk/books/366bday.htm`

Happy Birthday to You and You and You!

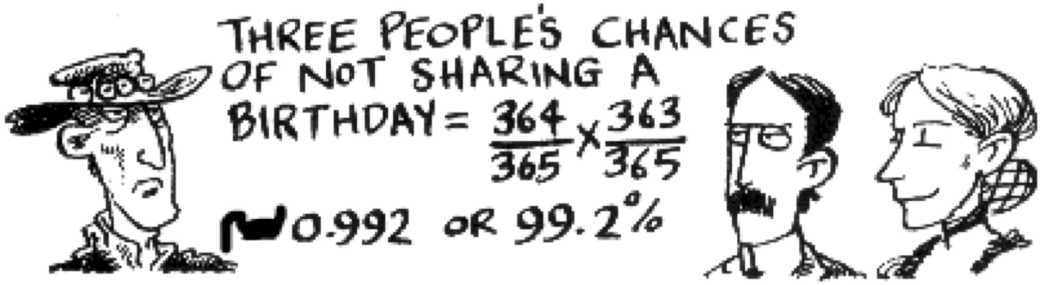
Happy Birthday to You and You and You!



Picture credit: `http://www.murderousmaths.co.uk/books/366bday.htm`

Probability of sharing a birthday?

Happy Birthday to You and You and You!

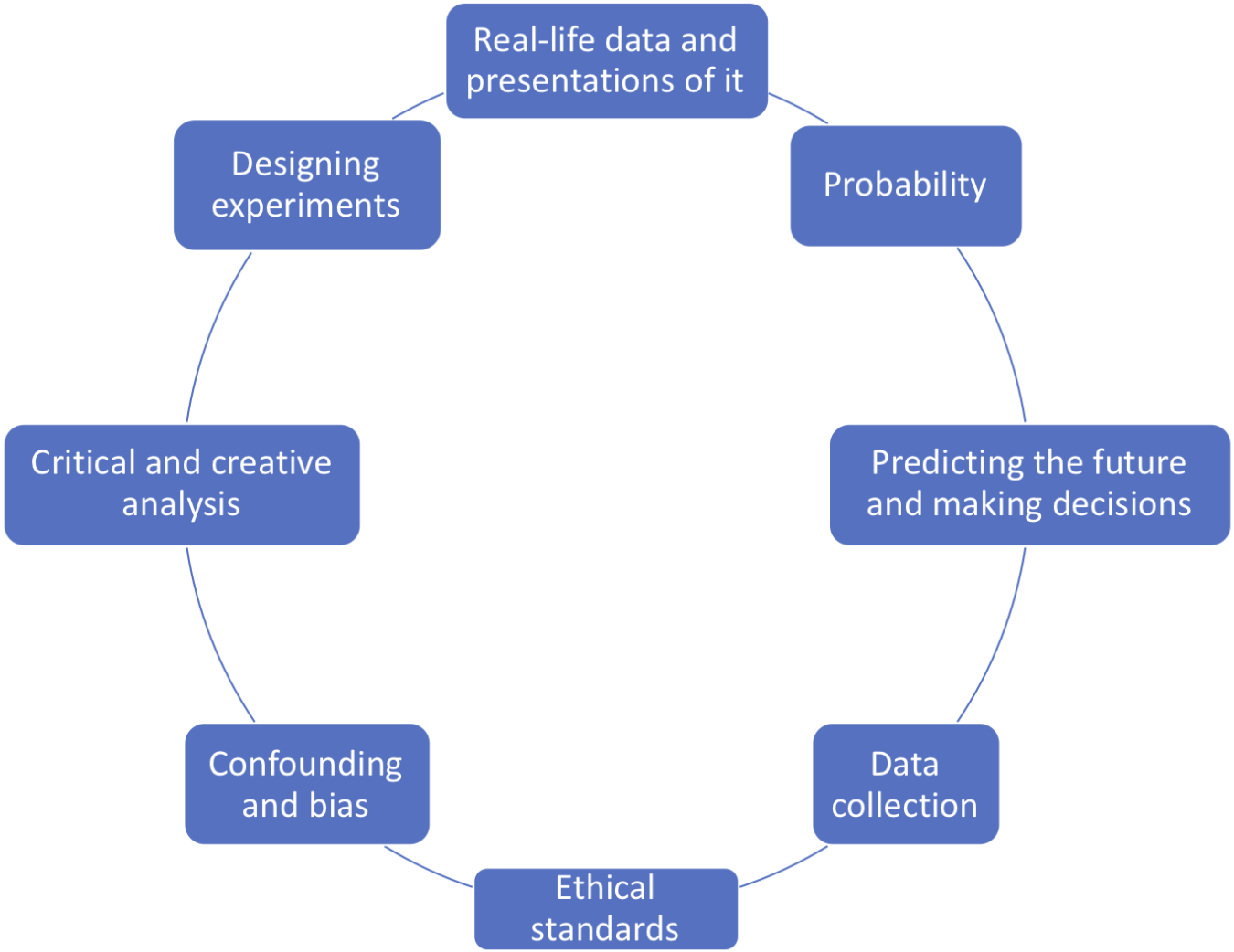


Picture credit: `http://www.murderousmaths.co.uk/books/366bday.htm`

Probability of sharing a birthday? 1- probability of NOT sharing

# people	approximate probability of two people with same birthday
2	.0027
3	.0082
5	.0271
20	.4114
23	.5073
25	.5687
50	.9704...

What are some factors that may invalidate independency/confounding variables for the birthdays?



What to Expect? Expected Value

If there is a 10% probability of rain, an organizer of an event could be worried this will impact it:

decision matrix:

	won't rain	will rain
probability	.90	.10
profit	+5000	-10,000

What to Expect? Expected Value

If there is a 10% probability of rain, an organizer of an event could be worried this will impact it:

decision matrix:

	won't rain	will rain
probability	.90	.10
profit	+5000	-10,000

expected value

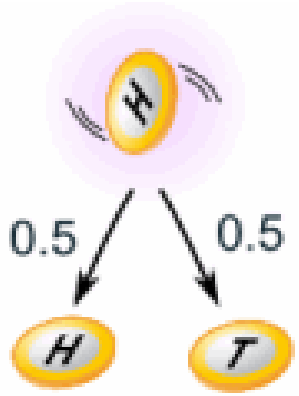
the weighted average of the probabilities and is often used in making predictions (and decisions)

= 5000(0.9) - 10000(0.1)

mean of repeating the experiment a large number of times


DR. RANDOM
PROFESSOR OF PROBABILITY
IN
OUT
PLEASE USE THIS COIN
sumanta.barua@gmail.com

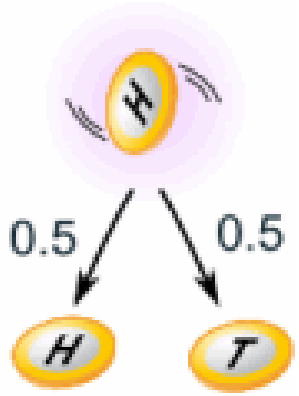
Coincidence or Expected?



Picture credit: `http://lriser03.blogspot.com/`

$$\frac{\text{number of outcomes for 4 heads}}{\text{total number of equally likely outcomes for all possibilities}}$$

Coincidence or Expected?

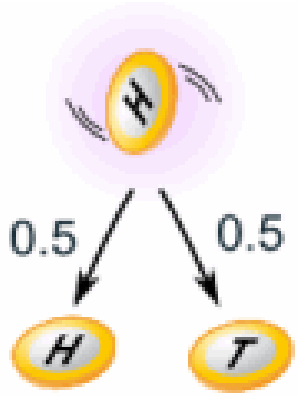


Picture credit: `http://lriser03.blogspot.com/`

$$\frac{\text{number of outcomes for 4 heads}}{\text{total number of equally likely outcomes for all possibilities}}$$

1 possibility for 4 heads: HHHH

Coincidence or Expected?



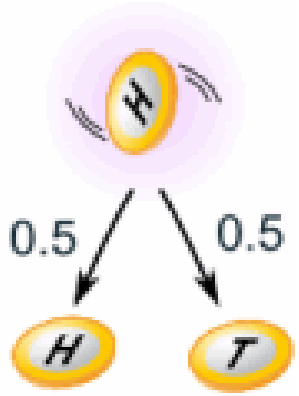
Picture credit: `http://lriser03.blogspot.com/`

$$\frac{\text{number of outcomes for 4 heads}}{\text{total number of equally likely outcomes for all possibilities}}$$

1 possibility for 4 heads: HHHH

How many possible outcomes total? 2 choices for each toss, so multiply $2 \times 2 \times 2 \times 2$

Coincidence or Expected?



Picture credit: `http://lriser03.blogspot.com/`

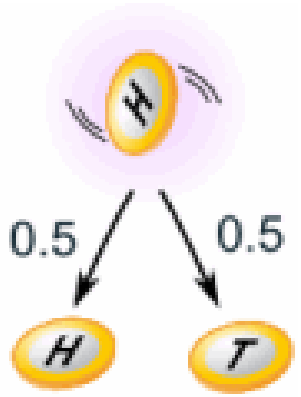
$$\frac{\text{number of outcomes for 4 heads}}{\text{total number of equally likely outcomes for all possibilities}}$$

1 possibility for 4 heads: HHHH

How many possible outcomes total? 2 choices for each toss, so multiply $2 \times 2 \times 2 \times 2$

probability of 4 heads in 4 tosses: $\frac{1}{16}$

Coincidence or Expected?



Picture credit: `http://lriser03.blogspot.com/`

$$\frac{\text{number of outcomes for 4 heads}}{\text{total number of equally likely outcomes for all possibilities}}$$

1 possibility for 4 heads: HHHH

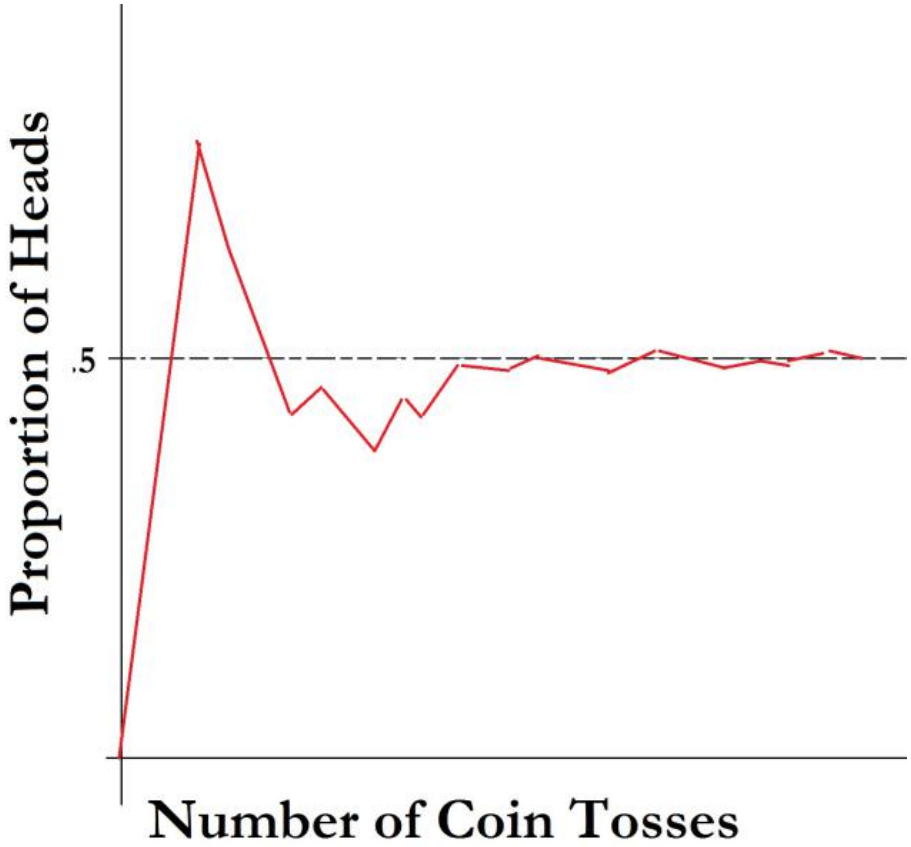
How many possible outcomes total? 2 choices for each toss, so multiply $2 \times 2 \times 2 \times 2$

probability of 4 heads in 4 tosses: $\frac{1}{16}$

expected number of people? $\frac{1}{16} \times$ number of people in class

Law of Large Numbers

- small number of experiments can have random fluctuations
- repeat an experiment a large number of times: outcome tends to the probability with much greater certainty



Picture credit: `http://0.tqn.com/d/statistics/1/S/T/0/-/-/lawoflargenumbers.jpg`

Expected Value Applications

$\geq$ .1”	$\geq$ 1”	$\geq$ 2”	$\geq$ 4”	$\geq$ 6”	$\geq$ 8”	$\geq$ 12”	$\geq$ 18”
58%	3%	0%	0%	0%	0%	0%	0%

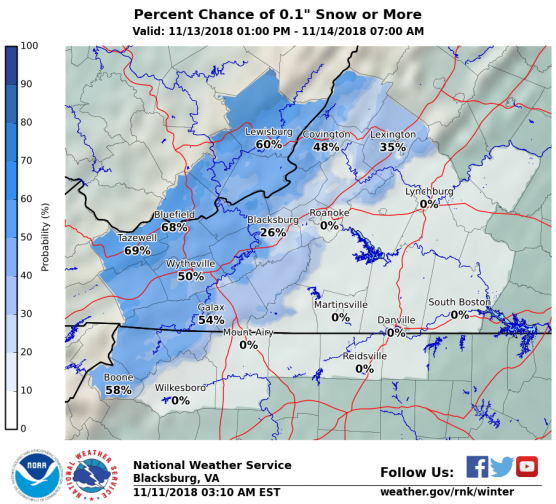


Image 1 and data: `https://www.weather.gov/rnk/winter`

Image 2: `https://www.pinnacle.com/en/betting-articles/Betting-Strategy/how-to-calculate-expected-value/EES2VE46TM4HTT32`

The weather, stocks and more are chaotic dynamical systems with uncertainty within expected values

Decision Matrix: Game Show Friend or Foe

Picture source: Logo of *Friend or Foe?* Copyright Game Show Network

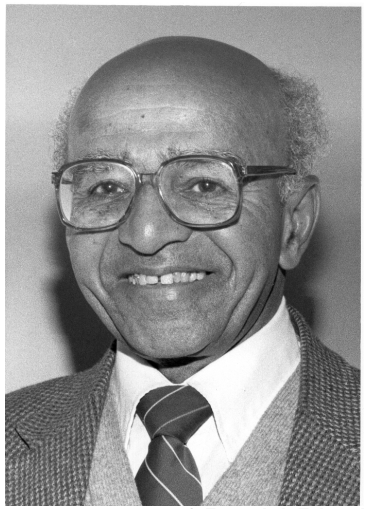
Contestants select whether the other person is a friend or a foe.

- If both choose friend, 50/50 split
- If both choose foe neither gets any money
- If one chooses friend and the other foe, then foe receives the entire trust fund and friend receives zero

	Contestant 2: Friend	Contestant 2: Foe
Contestant 1: Friend	$7500//$7500	0//$15000
Contestant 1: Foe	$15000//0	0//0

Activity from an article by William Butterworth and Paul Coe

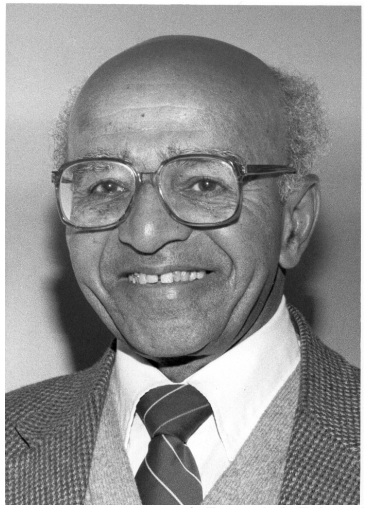
David Blackwell and Game Theory Controversy



Picture courtesy of David Blackwell

Basically, I'm not interested in doing research and I never have been... I'm interested in understanding, which is quite a different thing...

David Blackwell and Game Theory Controversy



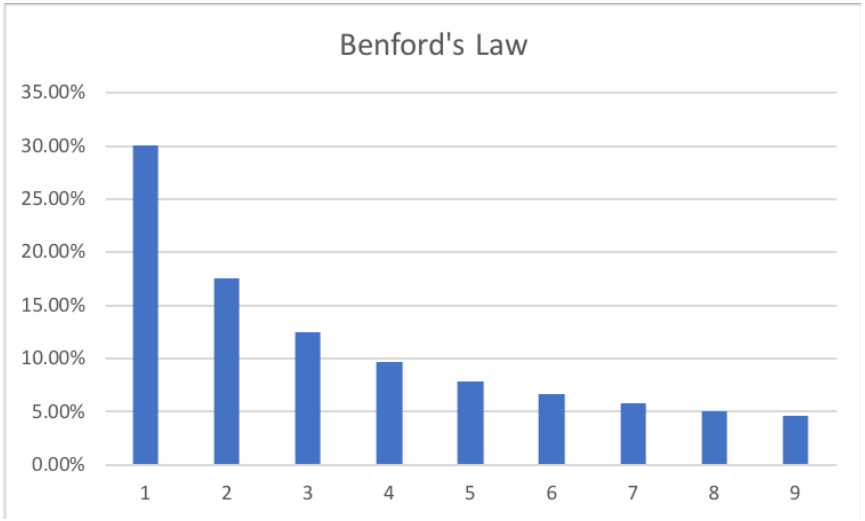
Picture courtesy of David Blackwell

Basically, I'm not interested in doing research and I never have been... I'm interested in understanding, which is quite a different thing... In fact, the situation with the Soviet Union has elements like this in it. To cooperate is to disarm and to double-cross is to re-arm with bigger and bigger weapons... So, when I saw that this led to an armaments race, so to speak, I realized I was not the one to come up with a satisfactory theory... [Albers & Alexanderson, 1985]

Interdisciplinary Perspectives and Academic Subjects

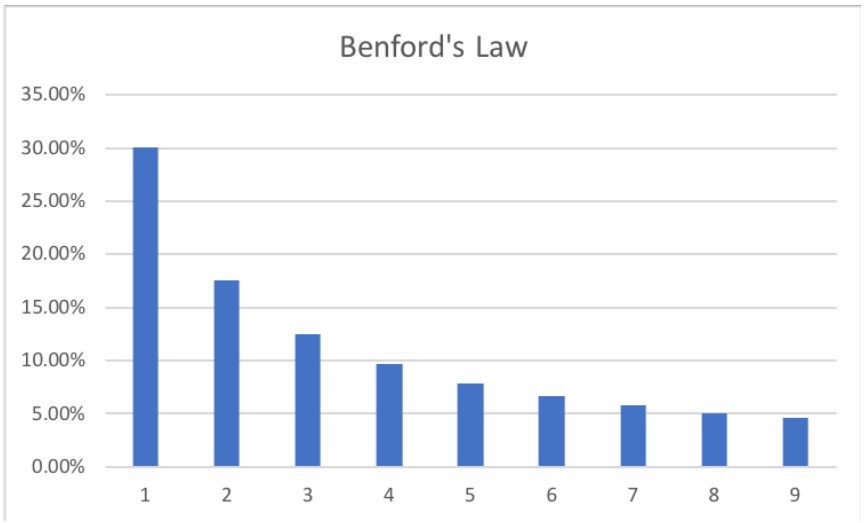
- Cooperative/selfish strategies: Tragedy of the Commons
- Ethics of what to work on
- Militarization of mathematics
- Prisoner's Dilemma
- Decision Matrices
- Game Theory
- Economics
- Business
- Environmental studies
- Psychology
- Statistics
- Mathematics
- Sports doping

Benford's Law: logarithmic trend in first digits



Frank Benford picture source: `http://www.nigrini.com/benfordslaw.htm`

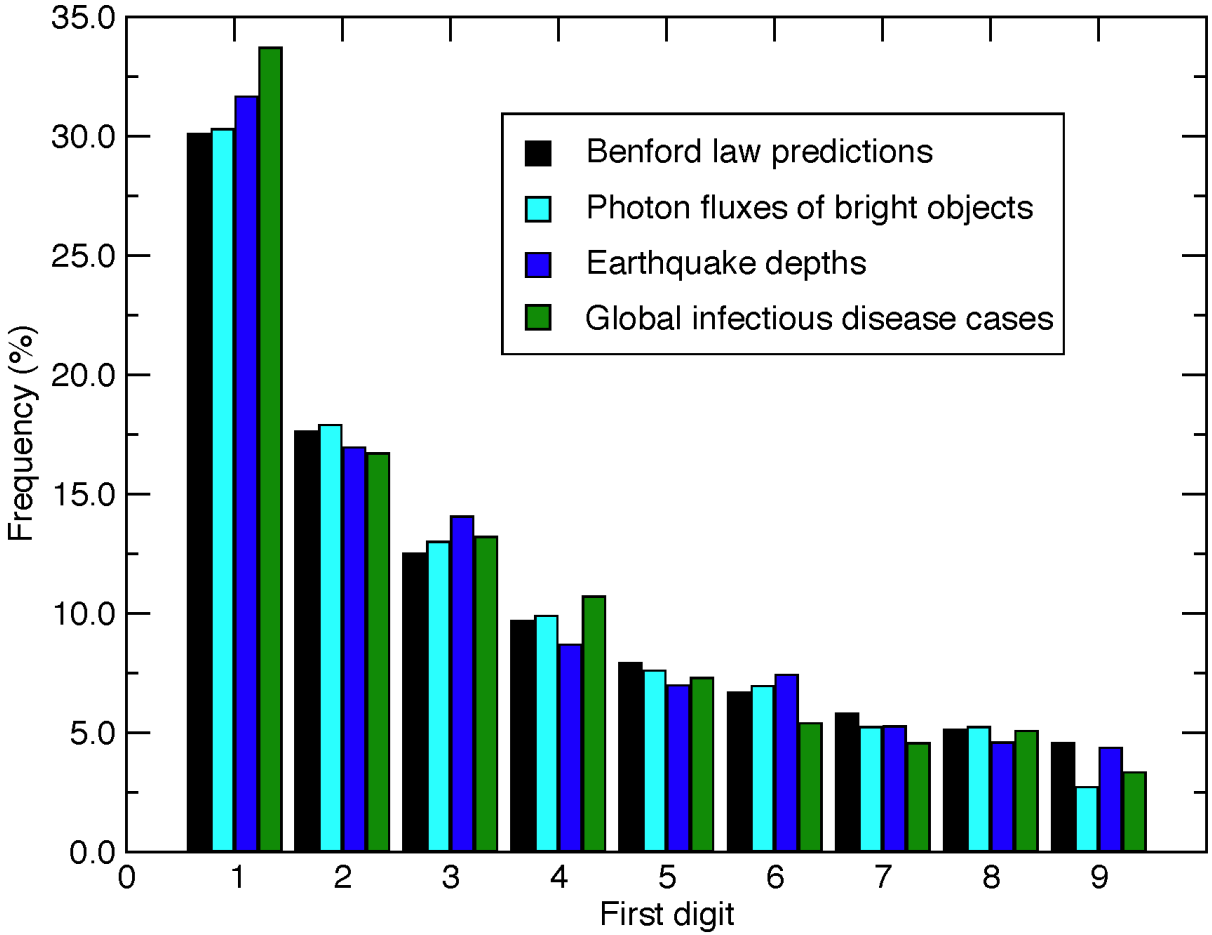
Benford's Law: logarithmic trend in first digits



Frank Benford picture source: `http://www.nigrini.com/benfordslaw.htm`

- Fraud detection of human or computer generated numbers—not definitive proof
- The larger the better, example > 500
- Not on data sets like the height of NBA basketball players

Portions adapted from "Using Excel and Benford's Law to detect fraud" by J. Carlton Collins *Journal of Accountancy*



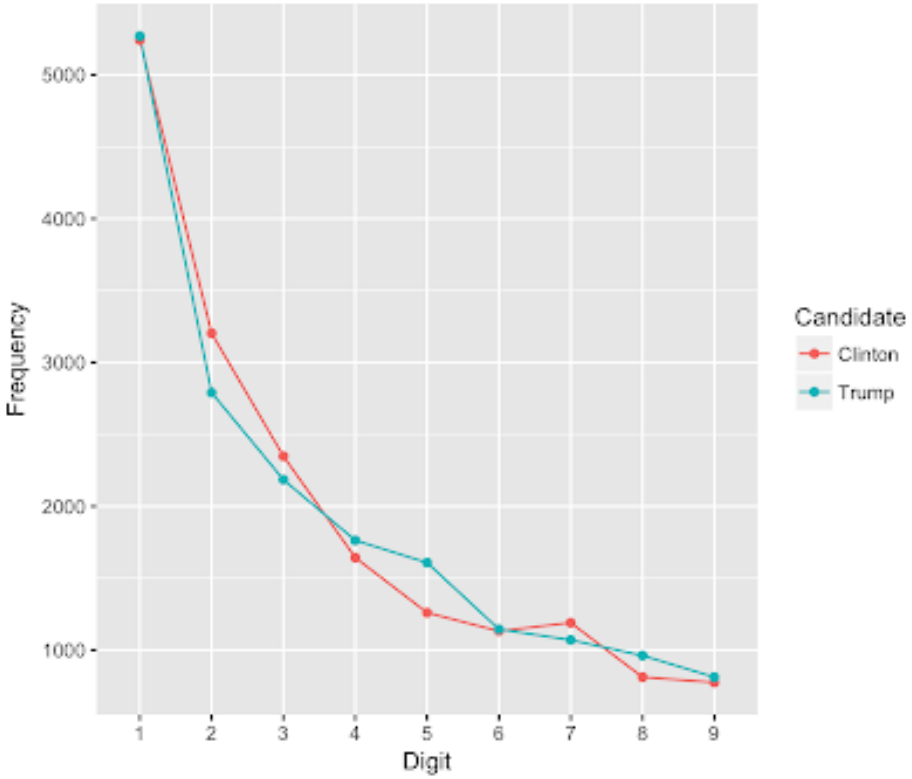
source: http://rses.anu.edu.au/highlights/view.php?article=109

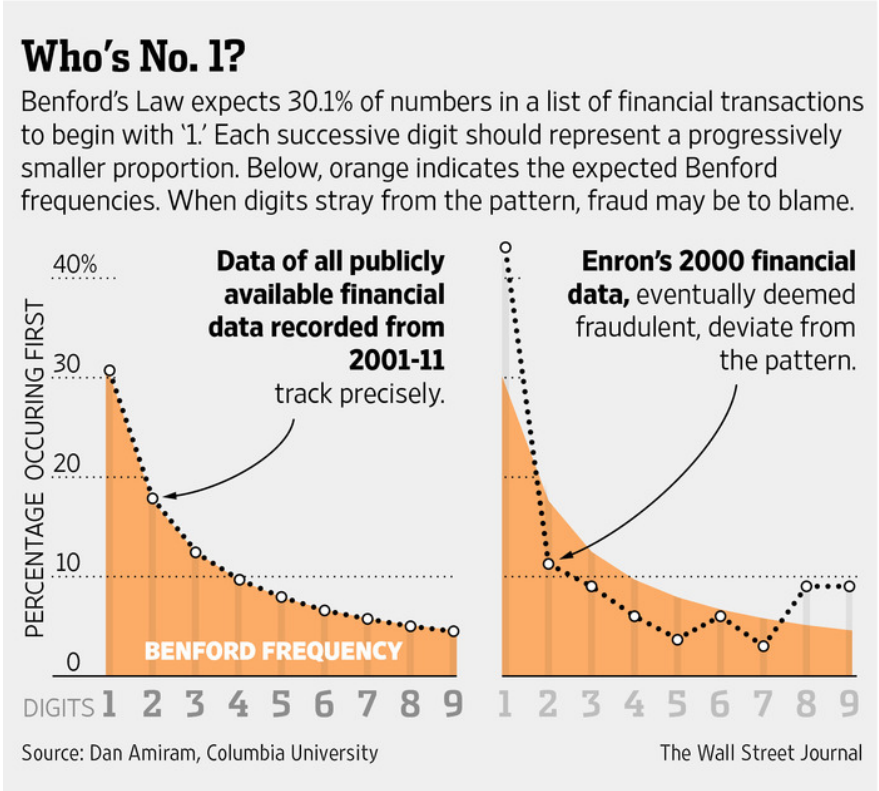
1452 bright objects identified by the Fermi space telescope

248915 global earthquakes—depths

987 reports of infectious disease World Health Organization

Benford's Law and Fraud Detection





overall county vote totals

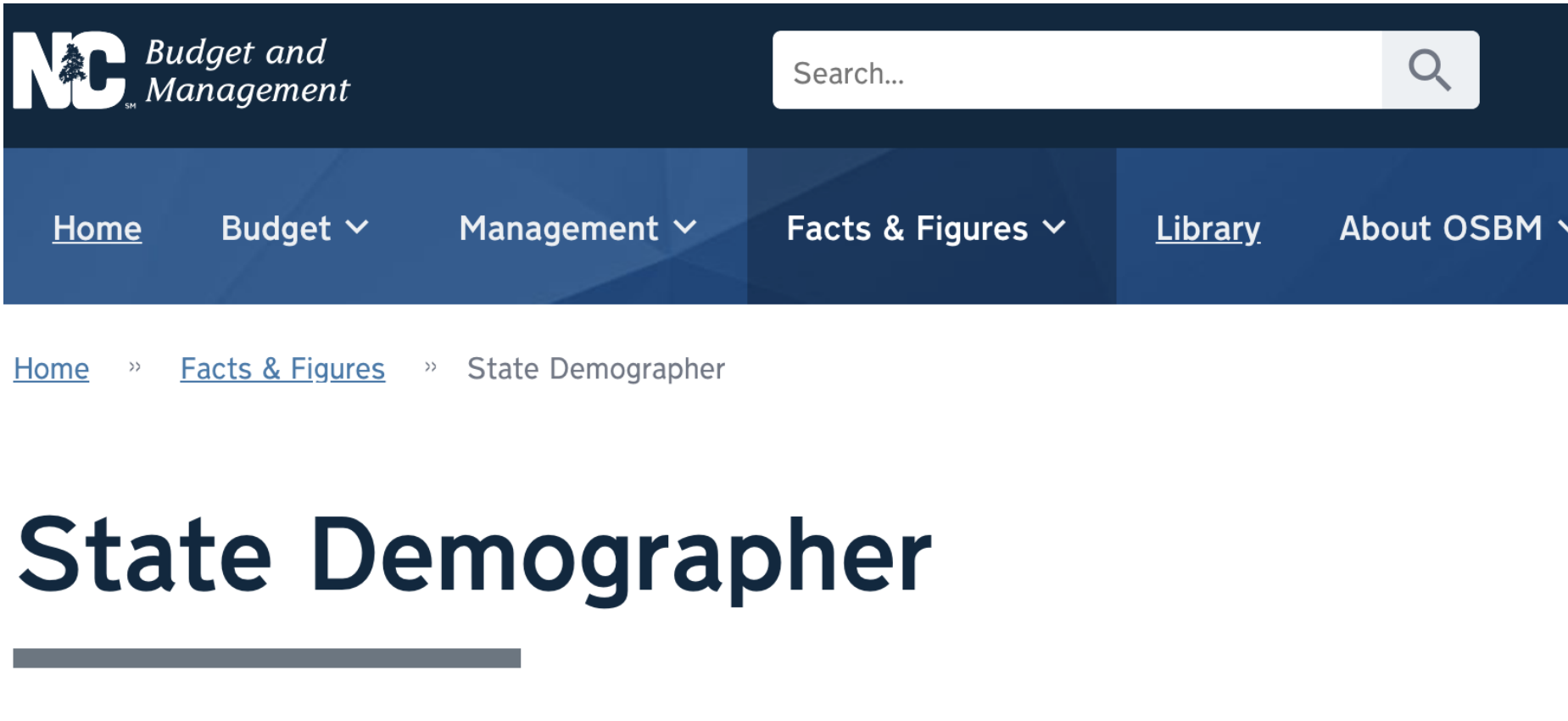
3,007 counties in the US

Picture credits:

`http://suehpro.blogspot.com/2016/12/election-results-vs-benfords-law-and.html`

`https://www.wsj.com/articles/accountants-increasingly-use-data-analysis-to-catch-fraud-1417804886`

North Carolina County Population Data



https://www.osbm.nc.gov/facts-figures/demographics

What to Expect?

What is the expected value of grades where percentages are
ASULearn 50%, class 5%, exams 30%, final project 15%
with averages of
ASULearn 85, class 100, exams 75, final project 95

a) 83
b) ~ 84
c) 85
d) other

The grading scale is: $A \geq 93; 90 \leq A- < 93; 87 \leq B+ < 90...$

The WORLD FACTBOOK

In "The WORLD FACTBOOK," the CIA lists Eswatini (a small landlocked country in southern Africa formerly known as Swaziland) as having the highest percentage of adults (aged 15–49) living with HIV/AIDS (`https://www.cia.gov/the-world-factbook/field/demographic-profile/` and `https://www.cia.gov/the-world-factbook/countries/eswatini/`, last updated 2023). The total population is 1,130,043 (2023 est.) and the percentage of people living with HIV is close to 28%.

Calculate how many people are HIV negative (round to the nearest whole person)?

a) 60
b) 316,412
c) 588,824
d) 813,631
e) other

Decision Matrix/Payoff Matrix

	Contestant 2: Friend	Contestant 2: Foe
Contestant 1: Friend	\$7500//\$7500	0//\$15000
Contestant 1: Foe	\$15000//0	0//0

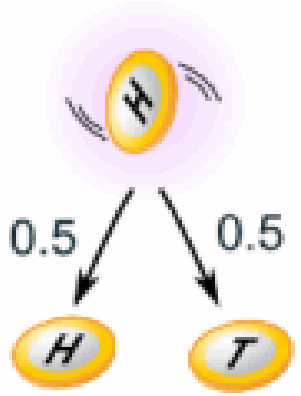
	Test+	Test-
Person is HIV +	HIV+ people $\times$ probability they test +	
Person is HIV -		
Total		

Would you support legislation for mandatory HIV testing?

Tossing Around

What is the probability of getting exactly 3 heads if you toss a fair coin 4 times? Assume independence.

a) .10
b) .25
c) .50
d) .75
e) other



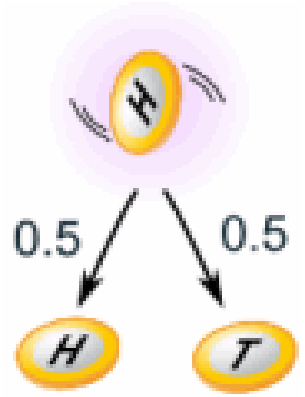
Picture credit: `http://lriser03.blogspot.com/`

$$\frac{\text{number of outcomes for 3 heads}}{\text{total number of equally likely outcomes for all possibilities}}$$

Tossing Around

What is the probability of getting exactly 3 heads if you toss a fair coin 4 times? Assume independence.

a) .10
b) .25
c) .50
d) .75
e) other



Picture credit: `http://lriser03.blogspot.com/`

$$\frac{\text{number of outcomes for 3 heads}}{\text{total number of equally likely outcomes for all possibilities}}$$

4 possibilities for 3 heads: THHH, HTHH, HHTH, and HHHT

How many possible outcomes total? 2 choices for each toss, so multiply $2 \times 2 \times 2 \times 2$

Expected Number of Courses

As of Fall 2023, ASU has approximately 21,253 students (`https://www.appstate.edu/about/`). The following table lists theoretical numbers of students registered by the number of courses they are taking. First fill in the table by computing the probabilities—number of registered / 21,253 Round to 2 decimals:

number of courses	1	2	3	4	5	6	7
number registered	213	638	2763	5313	8289	3613	425
probability	.01						

Next, compute the expected value of the number of courses by taking a weighted average of the number of courses (in row 1) and the probabilities (in row 3), such as $.01 \cdot 1 + ...$

Expected Number of Courses

As of Fall 2023, ASU has approximately 21,253 students (`https://www.appstate.edu/about/`). The following table lists theoretical numbers of students registered by the number of courses they are taking. First fill in the table by computing the probabilities—number of registered / 21,253 Round to 2 decimals:

number of courses	1	2	3	4	5	6	7
number registered	213	638	2763	5313	8289	3613	425
probability	.01						

Next, compute the expected value of the number of courses by taking a weighted average of the number of courses (in row 1) and the probabilities (in row 3), such as $.01 \cdot 1 + \ldots$

Even though these are theoretical registration numbers, do you think this reasonably approximates reality at Appalachian? Explain your reasoning.

Happy Birthday to You and You and You and You!

What is the probability to find a shared birthday in a group of 4 people? Assume independence and exclude February 29th.

a) $\sim .0164$

b) $\sim .5073$

c) $\sim .9836$

d) 4

e) other

TEDxBlackRockCity - Spencer Greenberg - Improve Your Life With Probability start with 1:42

https://www.youtube.com/watch?v=GZ69g8LtZc0