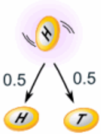
Role of chance and probability in real world situations

...helped bring mathematics into a more tangible thought process for me and gave further insight to how conceptual ideas connect to the world around us and our personal lives.

- quantitative measure of the likelihood of an event
- mathematical foundation of common sense and good judgment
- 0 to 1 (or 0% to 100%)
- law of large numbers
- experimental error provides an estimate of the inherent uncertainty associated with experimental procedures
- The probability of event E occurring =

$$\frac{\text{number of different outcomes in E}}{\text{total number of equally likely outcomes}}$$

If It Either Happens or It Doesn't (Independent Events)



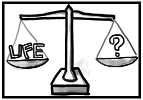
- probability that an event will happen =
  1 - probability it won't happen
- What is the probability of NOT rolling a 6 on a dice?
  $1 - \frac{1}{6} = \frac{5}{6} = \frac{\text{number of different outcomes}}{\text{total number of equally likely outcomes}} =$
  probability of rolling 1, 2, 3, 4 or 5.
- If a test is 95% accurate for people who have a disease then it correctly tests positive 95% of the time, but incorrectly tests negative for them (false negative) 5% of the time. *Sensitivity* is .95.
- If a test is 99% accurate for people who don't have a disease then it correctly tests negative 99% of the time, but incorrectly tests positive for them (false positive) 1% of the time. *Specificity* is .99.

Multiplication Rule for Independent Events

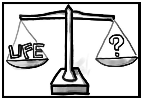
- If the probability of a person being left-handed is $\frac{1}{10}$, and the probability of being blue-eyed is $\frac{1}{3}$, then what is the probability of being left-handed and blue-eyed (assuming these are independent of each other)?

Multiplication Rule for Independent Events

- If the probability of a person being left-handed is $\frac{1}{10}$, and the probability of being blue-eyed is $\frac{1}{3}$, then what is the probability of being left-handed and blue-eyed (assuming these are independent of each other)?
- If independent, then the proportion of blue-eyed people among the left-handed people is the same as the proportion of blue-eyed people among the whole population, so
  left-handed and blue-eyed = $\frac{1}{3}$ of $\frac{1}{10}$ = $\frac{1}{3\times 10} = \frac{1}{30}$

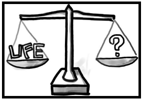


Let Test A be 95% accurate (sensitivity and specificity) and an independent Test B be 99% accurate. What is the probability that a positive person is actually infected?



Let Test A be 95% accurate (sensitivity and specificity) and an independent Test B be 99% accurate. What is the probability that a positive person is actually infected? Suppose the population of the US is p and that there are roughly $0.001p$ people with the disease.

- How many infected people will test positive?



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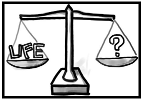
- How many infected people will test positive?
  $.95 \times .99 \times .001p = 0.0009405p$
- How many non-infected people will test + (false positives)?



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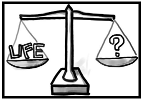
- How many non-infected people will test + (false positives)?
  $(1 - .95) \times (1 - .99) \times (p - .001p) = .05 \times .01 \times .999p = 0.0004995p$



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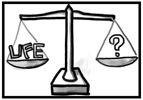
- How many infected people will test positive? $.95 \times .99 \times .001p = 0.0009405p$
- How many non-infected people will test + (false positives)? $(1-.95) \times (1-.99) \times (p-.001p) = .05 \times .01 \times .999p = 0.0004995p$
- Probability people with 2 + tests are actually infected? $0.0009405p/total$



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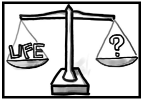
- Probability people with 2 + tests are actually infected?
  $0.0009405p/total =$
  $0.0009405p/(0.0009405p + 0.0004995p) = .653125.$



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- How many non-infected people will test + (false positives)?
  $(1 - .95) \times (1 - .99) \times (p - .001p) = .05 \times .01 \times .999p = 0.0004995p$
- Probability people with 2 + tests are actually infected?
  $0.0009405p/total =$
  $0.0009405p/(0.0009405p + 0.0004995p) = .653125.$
- If Test A is positive and Test B is negative, probability of infection = 0.00019.