Finding and Using Taylor Series Group Work Target Practice

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \text{ for all x}$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots \text{ for all x}$$
$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \text{ for all x}$$
$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots \quad \text{ for } |x| < 1$$

Composition 1: Use a known Taylor series to find the Taylor series about 0 for $\sin(2x^3)$ (Hint: substitute i.e/ composition of functions into the series for sin).

Composition 2: Use a known Taylor series to find the Taylor series about 0 for $\frac{1}{1+x^2}$. (Hint: substitute $-x^2$ for x in the geometric one, since this looks like that)

The radius of convergence is inherited from the original series, so what is the radius of convergence?

Differentiation: We can take derivatives of Taylor series: just take the derivative term by term. The radius of convergence of the derivative will be the same as that of the original series. To see this, take the derivative of the Taylor series of the sine function, term by term, to get the Taylor series of the cosine function. Reduce to show that you get the usual Taylor series of cosine.

Integration: It is also possible to integrate Taylor series term-by-term. Use a known Taylor series to find the Taylor series about 0 for $\arctan x$. (Hint: first, what is the derivative of $\arctan x$—look for that Taylor series above in what we computed. Next integrate term by term.)

What is the radius of convergence?

Historically, this was used to estimate digits of π!

Estimating an Integral: We can use some of the terms to estimate a number like π, but we can also integrate Taylor terms to estimate an integral that we cannot compute otherwise.

Find the first three terms of the Taylor series for $\sin(x^2)$ by using a known Taylor series.

Find the first three terms of the Taylor series for $\int \sin(x^2)\ dx$, which is an integral that is not elementary!

To find an estimation of $\int_0^1 \sin(x^2)\ dx$, evaluate the response from Part b) at the endpoints of the interval (no need to simplify).

Product: We can also find the Taylor series of a product of two Taylor-expandable functions. Apply this idea for the function $h(x) = e^x \cos x$. Find the degree 3 Taylor polynomial. Foil to find all the terms where the resulting power (using exponent rules) is less than or equal to 3.

For your notes: Lagrange Error Bound/Taylor's Inequality problem solving steps to help you in WileyPlus

1. What is $n + 1$?
2. Compute the $(n + 1)^{\text{st}}$ derivative of $f(x)$
3. Write the absolute value of the $(n + 1)^{\text{st}}$ derivative of $f(x)$ (get rid of anything negative).
4. Can you tell whether the absolute value of the $(n + 1)^{\text{st}}$ derivative increasing or decreasing on the interval?
5. Find M as a natural upper bound on the absolute value of the $(n+1)^{\text{st}}$ derivative on the interval and write down the bound and your reasoning. Either use increasing/decreasing reasoning, or check the endpoints if it is a monotone function.
6. Fill in the various components from the Taylor polynomial error on the Series Theorems sheet, but do not simplify.
7. (Only if directed) Compute the Lagrange error bound/Taylor inequality on your calculator.