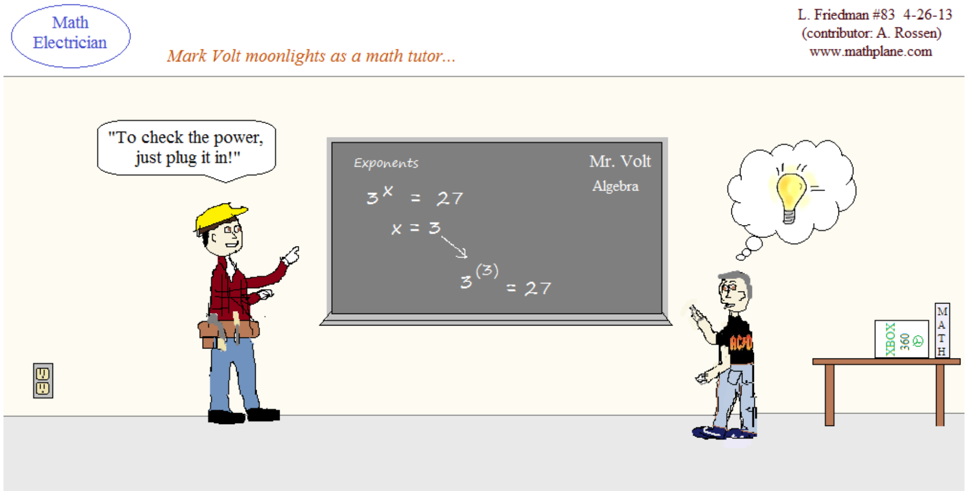
11.1 Testing DEs

11.1 really does come down to this math comic:



Alex gets the connection!

1. You pour a cup of coffee at 180° and so Newton's law of cooling applies. Let T be the temperature and t be time. Then Newton's law of cooling specifies that the differential equation is $\frac{dT}{dt} = -k(T-72)$, where 72° is the temperature of the room. Is $T(t) = 72 + 108e^{-kt}$ a solution to the differential equation?

   We check by plugging in to $\frac{dT}{dt} = -k(T-72)$—we'll need the derivative of $T(t) = 72 + 108e^{-kt}$ for the left side, so use chain rule: $T'(t) = 0 + 108e^{-kt}(-k)$. Now we plug in the derivative to the left side and the proposed solution to the right side, and we see if they satisfy the differential equation:
   $108e^{-kt}(-k) = \frac{dT}{dt} \stackrel{?}{=} -k(T-72) = -k(72 + 108e^{-kt} - 72) = -k108e^{-kt}$.
   These are equal, so it is a solution.

   Notice that 108 actually comes from the difference between the coffee and room temperature: 180-72 = 108. These form the basis of many real-life growth and decay problems, including Newton's law of cooling and population modeling, to name a few.

2. Which of the following is a solution to $y' = ky$? $y(t) = e^{kt}$, $y(t) = e^{kt+5} = e^{kt}e^5$, $y(t) = 2e^{kt}$?

   We check by plugging each proposed solution into $y' = ky$ to see if it works–we'll need the derivative for the left side, so we'll use chain rule:

   a) $y(t) = e^{kt}$ $\quad ke^{kt} = y' \stackrel{?}{=} ky = ke^{kt}$. Yes it is a solution.

   b) $y(t) = e^{kt+5} = e^{kt}e^5$ $\quad ke^{kt+5} = y' \stackrel{?}{=} ky = ke^{kt+5}$. Yes it is a solution.

   c) $y(t) = 2e^{kt}$ $\quad k2e^{kt} = y' \stackrel{?}{=} ky = k2e^{kt}$. Yes it is a solution.

3. Do either of $y(t) = e^{kt+2}$ or $y(t) = 2e^{kt}$ work when $y(0) = 2$?

   We saw above that each of these are solutions, but the question here is whether they solve the given initial condition. Since $y(0) = 2$, we plug in t=0 and see whether y=2:

   a) $y(t) = e^{kt+2}$ $\quad y(0) = e^{k0+2} = e^2 \neq 2$, so this is not a solution for $y(0) = 2$.

   b) $y(t) = 2e^{kt}$ $\quad y(0) = 2e^{k0} = 2e^0 = 2 \cdot 1 = 2$, so yes this is a solution for this initial condition.