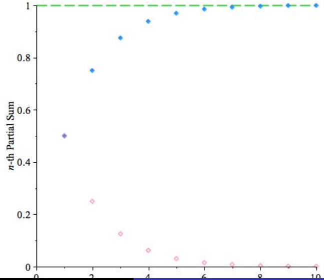
9.2 Geometric Series Review

- Geometric Series a= starting term, x=constant ratio of each term to preceding one
  $\sum_{i=0}^{\infty} ax^i = \frac{a}{1-x}$ when $|x| < 1$ and diverges otherwise

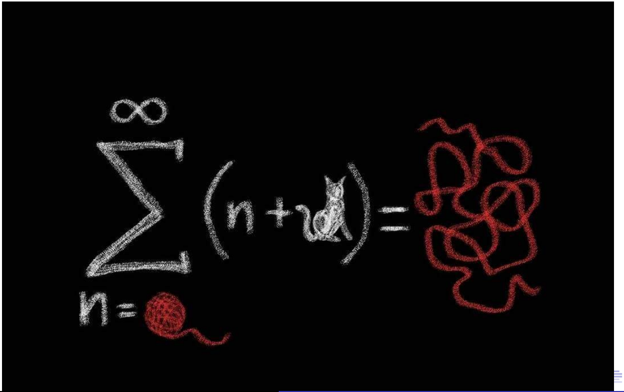
n^{th} partial sum (1st n terms added): $\sum_{i=0}^{n-1} ax^i = \frac{a(1-x^n)}{1-x}$ for $x \neq 1$

Example: $\sum_{i=0}^{n-1} \frac{1}{2}\frac{1}{2}^i = \sum_{i=1}^{n} \frac{1}{2}^i$ careful of starting # and index



9.3: Terms Not Getting Smaller

- terms not getting smaller: $\lim_{n\to\infty} a_n \neq 0$ or DNE, then partial sums diverge and so does the series. Example: $\sum_{n=1}^{\infty} \frac{5+n}{2n+1}$
  CAUTION: If $\lim_{n\to\infty} a_n = 0$ then the test is inconclusive and we must select another test.



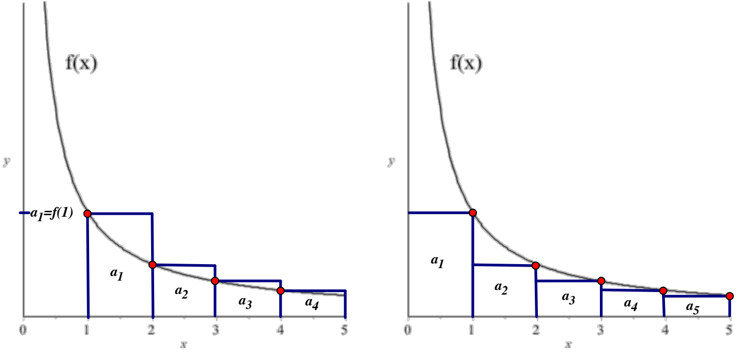
9.3:Linearity for Convergence or Divergence

- Linearity: $\sum_{n=1}^{\infty} a_n$ converges to S and $\sum_{n=1}^{\infty} b_n$ converges to T, and k is any constant, then $\sum_{n=1}^{\infty} ka_n + b_n$ converges to $kS + T$.

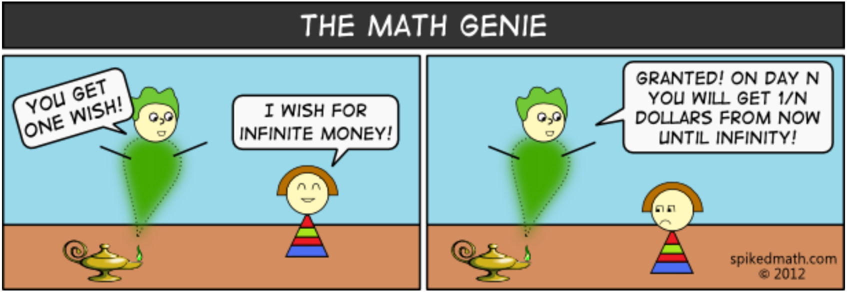
  Application 1: add two geometric series (converge to sum)
  Application 2: add divergent & convergent series (diverge)
  Example: $\sum_{n=1}^{\infty} \frac{1}{2}^n + (-1)^n$.
  Diverges, because if it were convergent, then subtract convergent $\sum_{n=1}^{\infty} \frac{1}{2}^n$ and the result should converge by linearity, but doesn't!

9.3: Integral Test Bounds

If series has terms that are decreasing and positive, the integral test not only tells us about convergence, but also bounds the series:



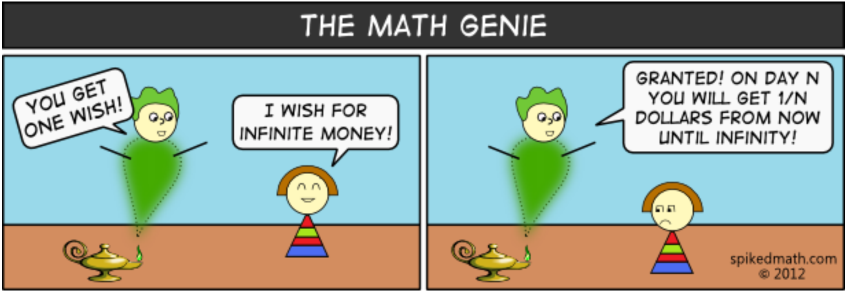
$$\int_1^\infty f(x)dx \leq \sum a_n \leq a_1 + \int_1^\infty f(x)dx$$



Harmonic series $\sum_{N=1}^{\infty} \frac{1}{N}$ diverges by growing to ∞ slowly! Why?

Integral Test

$$\int_1^\infty \frac{1}{x}dx = \lim_{b\to\infty} \int_1^b \frac{1}{x}dx =$$



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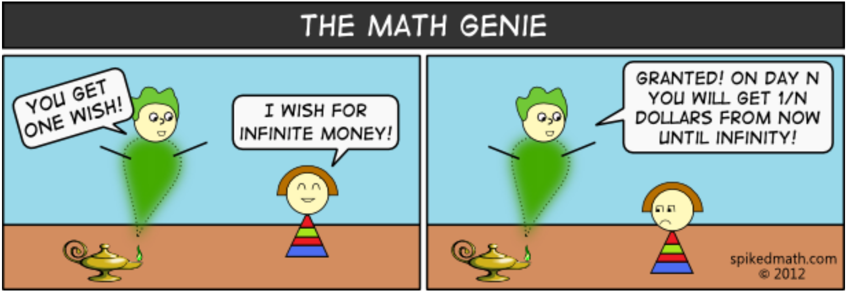
$$\int_1^{\infty} \frac{1}{x} dx = \lim_{b\to\infty} \int_1^b \frac{1}{x} dx = \lim_{b\to\infty} \ln(x)\Big|_1^b =$$



Harmonic series $\sum_{N=1}^{\infty} \frac{1}{N}$ diverges by growing to ∞ slowly! Why?

Integral Test

$$\int_1^{\infty} \frac{1}{x} dx = \lim_{b \to \infty} \int_1^{b} \frac{1}{x} dx = \lim_{b \to \infty} \ln(x)\Big|_1^b = \lim_{b \to \infty} \ln(b) - \ln(1)$$

diverges so series does too

9.3: Integral Test

- For $\sum_{1}^{\infty} a_n$, if the terms are decreasing and $a_n > 0$ then the series behaves the same way as $\int_{1}^{\infty} a_n dn$.
  So look for decreasing and positive terms (eventually) that we can integrate (Calc I or Chap 7) + improper integral. Otherwise the test does NOT help.