Applications of Differential Equations

Throw a ball in the air—where does it go? Put a pot on the stove—how does the heat spread? Blow a soap bubble—what shape will it take? Flip a switch and... a light turns on! All of these phenomena are governed by simple differential equations that have been known and studied for over 100 years. Nature at a small scale—electrons, protons and other particles—is governed by more complicated laws; these also involve differential equations. [Karen Uhlenbeck, Mathematics: The Equations of Nature]

$$\frac{dy}{dt} = ky$$

Many objects grow and shrink proportional to the amount present

- Glacier
- Exponential growth at a continuous rate
- Compound interest: $\frac{dB}{dt} = rB$, where r is the interest rate, or other economic systems like those modeling labor
- Drug metabolism: $\frac{dA}{dt} = kA$, where k is negative
- Pollution in a lake: $\frac{dQ}{dt} = -r\frac{Q}{V}$, where r is the rate of flow, V is the volume of a lake, and no new pollution is introduced
- Snowball melts rate proportional to surface area (write r in terms of V)
- Sometimes all we know about a given system is its rate of change

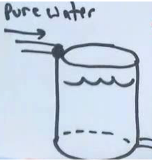
$$\frac{dT}{dt} = -k(T - \text{surrounding temperature}), k > 0$$

Heating and Cooling

- cooling: coffee, object in refrigerator, deceased body in forensic science...
- warming: cooking in oven...

$$\frac{dy}{dt} = \text{rate1} - \text{rate2}$$

- mixing problems: rate of salt in - rate of salt out
- population: birth rate - death rate
- weight : pounds/calorie (intake - maintenance)



http://www.hotwebclips.com/Home/Show/JplHqzyuYf8

Newton's Law of Cooling

You pour a cup of coffee at 180°. Assume that 72° is the temperature of the room.

Define variables and write the DE and the initial condition:

differential equation __________ initial condition ______

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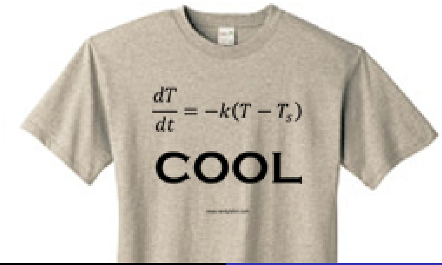
T temperature of the cooling coffee, t be time, and $k > 0$
DE: $\frac{dT}{dt} = -k(T - 72)$ by Newton's law of cooling
initial condition: $T(0) = 180$

http://www.nerdytshirt.com/cool.html



Growth and Decay: Glacier

- Early stages, populations may change rapidly at a rate proportional to the current population
  Example: In 2007 Grinnell Glacier in Glacier National Park covered about 142 acres and was shrinking exponentially at a rate of 4.3% of it's acreage each year.
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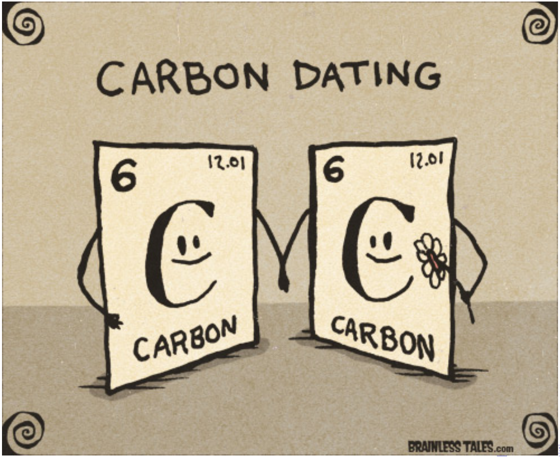
- If the model continues to hold, when will the glacier be half of it's 2007 size? When will it effectively disappear?

https://trailheadtraveler.com/2017/12/best-day-hikes-in-glacier-national-park/

Carbon-14 Dating

- Approximately what percentage of its carbon-14 should remain in a painting by Johannes Vermeer (1632–1675)? The half-life of carbon-14 is about 5730 years.

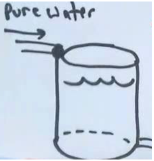


Team led by Willard F. Libby developed radiocarbon method. Nobel Prize in Chemistry in 1960: *for his method to use Carbon-14 for age determinations in archaeology, geology, geophysics, and other branches of science*

Image: `https://media1.britannica.com/eb-media/29/21029-004-7FBDB267.jpg`

Mixing Problems

A tank contains 100L of brine with a concentration of 0.2 kg of salt per liter (i.e. .2 kg/L $\times$100 L = 20 kg salt). Pure water enters the tank at a rate of 10 L/min and the resulting solution, which is stirred continuously, runs out at the same rate.



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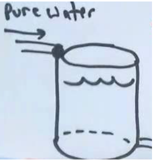

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  S kg salt, t minutes
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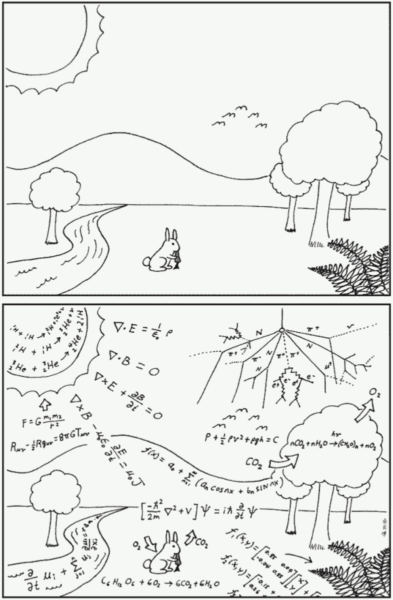
Image: http://www.hotwebclips.com/Home/Show/JplHqzyuYf8



This is how scientists see the world.

https://abstrusegoose.com/strips/all_i_see_are_equations.PNG

DEs and PDEs: Existence and Uniqueness of Solutions?

Differential equations can be very hard to solve. In fact, a large number of them can't be solved at all! At least not numerically. Some qualitative information about the overall behavior of the system can sometimes still be learned—this is the field of study of dynamical systems [Karen Uhlenbeck, Mathematics: The Equations of Nature]

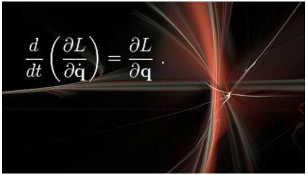


Image: `https://www.shutterstock.com/g/marcpk`

Euler-Lagrange equations: L measures energy in a physical system

Emmy Noether's theorem (roughly): Symmetric systems have corresponding conservation laws