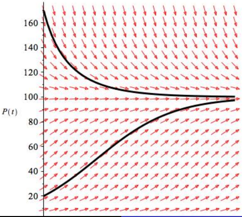
11.7 Logistic Growth: northern spotted owl

- If they have plenty of food and space, then their population would grow proportional to the number of owls around to reproduce, i.e., exponentially. But the owls are territorial and reproduce less often if they don't have sufficient space.

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right), P(0) = P_0$$

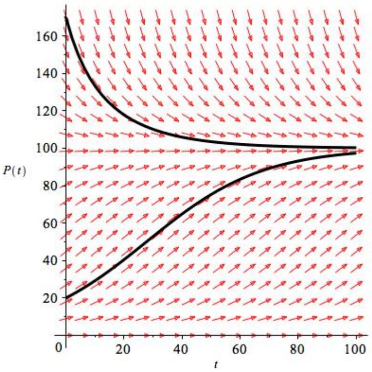
  M is the maximum sustainable population
- P is small—close to exponential growth
- P closer to M, $\frac{dP}{dt}$ slows.



Logistic Growth Solution

$$\frac{dP}{dt} = 1.02P\left(1 - \frac{P}{5000}\right)$$

$$P(t) = \frac{5000}{1 + Ce^{-1.02t}}$$



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$1 = A \cdot P + B \cdot (1 - \frac{P}{5000})$ so $1 = B + AP - \frac{B}{5000}P$

$B = 1$ $P(A - \frac{B}{5000}) = 0$, so $A = \frac{B}{5000} = \frac{1}{5000}$

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$$\frac{dP}{P\left(1 - \frac{P}{5000}\right)} = 1.02dt$$

$$\frac{\frac{1}{5000}}{1 - \frac{P}{5000}}dP + \frac{1}{P}dP = 1.02dt$$

$$-\ln\left|1 - \frac{P}{5000}\right| + \ln|P| = 1.02t + C$$

Logistic Growth Solution: Solving for P

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Using log rules, the left side natural logs can be combined as:

$$\ln\left(\frac{P}{1 - \frac{P}{5000}}\right) = 1.02t + c$$

$$\frac{P}{1 - \frac{P}{5000}} = e^{1.02t+c}$$

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Next, $P = e^{1.02t+c} - e^{1.02t+c}\frac{P}{5000}$, so $P(1 + \frac{e^{1.02t+c}}{5000}) = e^{1.02t+c}$
Then $P(t) = \frac{e^{1.02t+c}}{1+e^{1.02t+c}/5000}$

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Mathematicians would not leave the fraction in the denominator, so we will multiply by $5000/5000$ and factor out the exponentials from the numerator and denominator:

$$P(t) = \frac{5000e^{1.02t+c}}{5000 + e^{1.02t+c}} = \frac{5000}{5000e^{-(1.02t+c)} + 1}$$

Lastly, we split the exponential function using the fact that $e^{-(1.02t+c)} = ce^{-1.02t}$ & put the 5000 and c constant together:

$$P(t) = \frac{5000}{1 + 5000ce^{-1.02t}} = \frac{5000}{1 + Ce^{-1.02t}}$$

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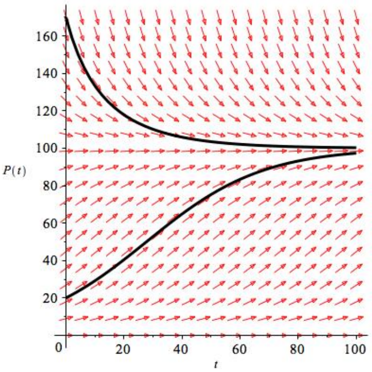
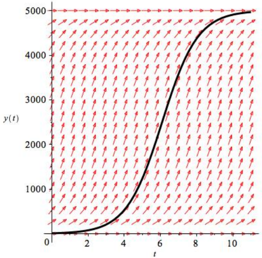


Bird flu H1N2

- 5000 bird flock, P(0)=10, and in the early stages k=1.02.

$$\frac{dP}{dt} = 1.02P\left(1 - \frac{P}{5000}\right), P(0) = P_0$$

- Estimate when the infection is spreading most quickly



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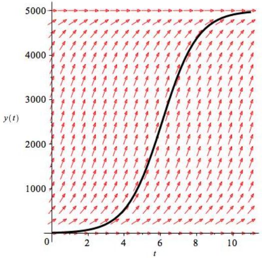

- What is the solution to the DE?
- How many birds are infected after 2 days, 5 days?
- When are 50% of the birds infected? 90%?