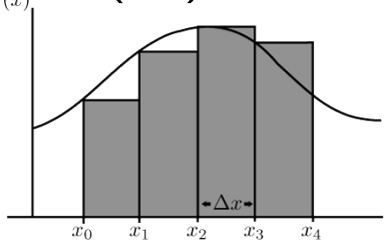
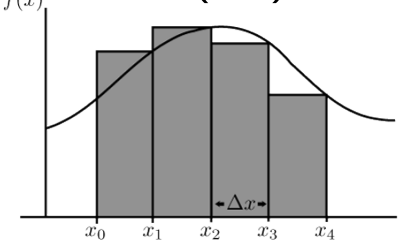
7.5 Numerical Methods

- Approximates integrals we can't evaluate directly, including discrete data
- n= number of intervals, $\triangle x = \frac{b-a}{n}$, $x_{i+1} = x_i + \triangle x$
- $Left(4) = f(x_0)\triangle x + f(x_1)\triangle x + f(x_2)\triangle x + f(x_3)\triangle x$ left endpoints
- $Right(4) = f(x_1)\triangle x + f(x_2)\triangle x + f(x_3)\triangle x + f(x_4)\triangle x$ right points

left:



right:



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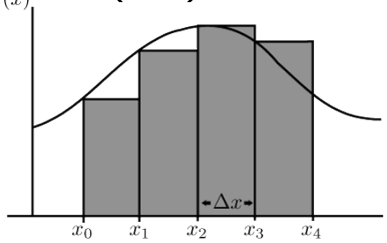
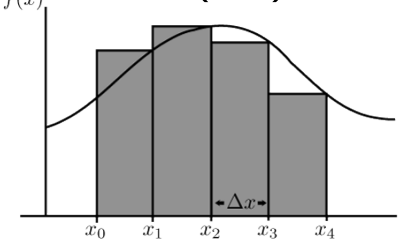
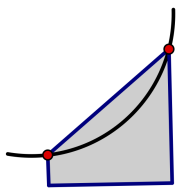


right:

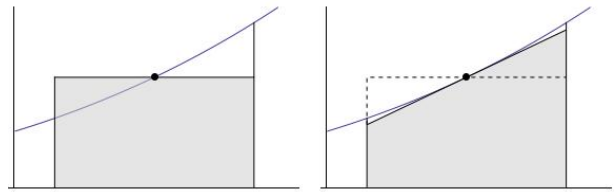


- $Trap(4) = \frac{Left(4)+Right(4)}{2}$ connect left and right points
- $Mid(4) = f(\frac{x_0+x_1}{2})\triangle x + f(\frac{x_1+x_2}{2})\triangle x + f(\frac{x_2+x_3}{2})\triangle x + f(\frac{x_3+x_4}{2})\triangle x$ midpoints

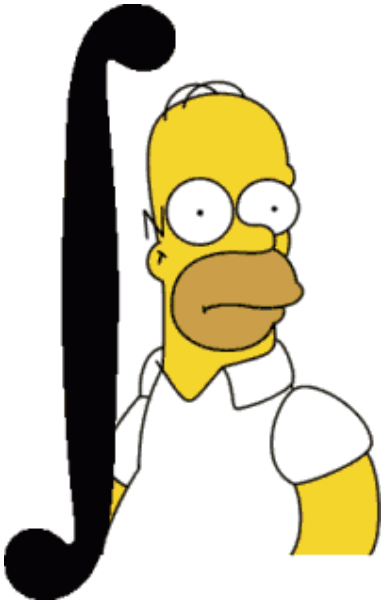
trapezoid:



midpoint:



Simpson's Rule



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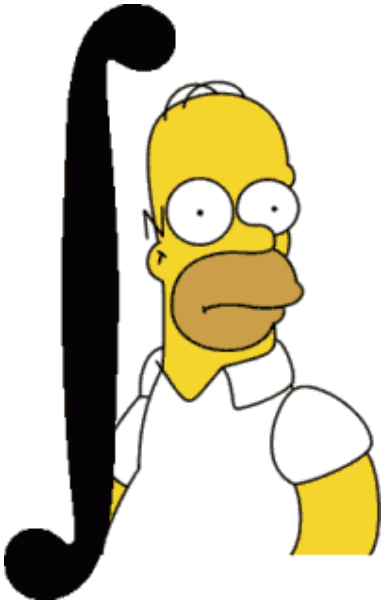

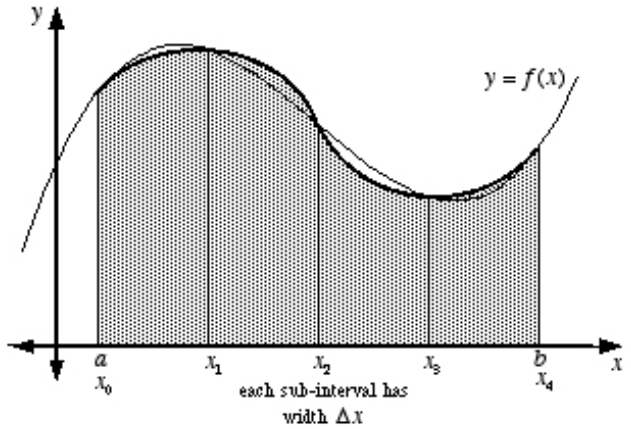

The shaded area bounded by the parabolas (the thicker curves) is approximately equal to the area bounded by $y = f(x)$.

$$\int_a^b f(x)dx = \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$$

- No not that Simpson!
- Thomas Simpson (1710–1761)

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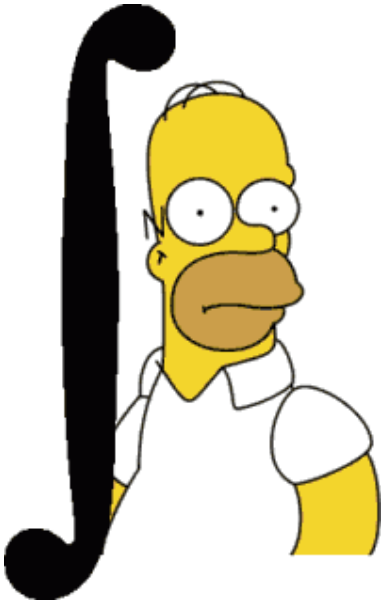

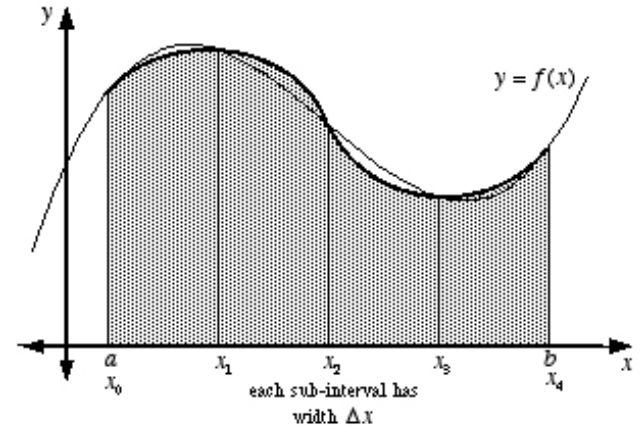

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- Johannes Kepler (1571–1630), volume of a wine barrel
- $\frac{2Mid(n)+Trap(n)}{3}$
- fits parabolas

Clicker Question

1. Which is true for $y = x^2$ when $x \geq 0$?

a) *Right*(n) and *Trap*(n) give overestimates

b) *Left*(n) and *Mid*(n) give overestimates

c) *Right*(n) and *Mid*(n) give underestimates

d) *Left*(n) and *Trap*(n) give underestimates

Clicker Question

2. Which is true for $y = e^{-x}$ when $x \geq 0$?

a) $Mid(n) \leq \int f(x)dx \leq Trap(n)$

b) $Trap(n) \leq \int f(x)dx \leq Mid(n)$

c) it depends on n

d) no way to tell

e) other

Even More History and Applications

- Many applications do not have a closed form, so numerical approximations are needed
- 2016 analysis–Babylonians used trapezoids under curve for Jupiter's speed over time. Area approximates degrees of movement

- 14th century in Europe