7.2 Parts (Undoing the Product Rule)

7.1 Substitution (Undoing the Chain Rule)

- Often w "inside" of some other function, dw up to constant

What I want you to show me... w, dw, $\int$ with respect to w

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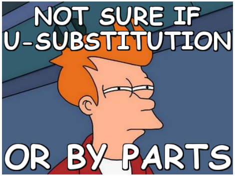

Clicker Question

1. For which of the following integrals is integration by parts with $u = x$ and $v' =$ (the rest of the integrand) a reasonable choice?

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For parts, often look for products mixing
$x^k, e^x, sin(x), cos(x), ln(x), arctan(x), arcsin(x)$
but not $ln(x)$ with $\frac{1}{x}$
nor $arctan(x)$ with $\frac{1}{1+x^2}$
nor $arcsin(x)$ with $\frac{1}{\sqrt{1-x^2}}$
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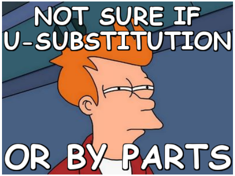

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Is the integral a *w*-subs, parts, both, or neither?

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- $\int e^{-x^2} dx$ not elementary, later we'll see numerical methods and Taylor series approximations

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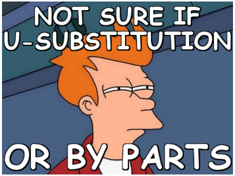

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2. For which of the following integrals is integration by parts a reasonable choice?

a) $\int x^{10} \ln x \, dx$

b) $\int \sin x^2 \, dx$

c) both of the above

d) none of the above

History and Applications

Integration by Parts is attributed to Brook Taylor (1685-1731)

- Deriving the Euler-Lagrange equation—how a physical system evolves through time from Hamilton's Least Action Principle
- CRC Handbook of Chemistry and Physics
- Engineering
- Journal of Geology and Geophysics. Earthquakes
- Image processing
- ... integrals made up of function products

When in doubt, integrate by parts [Micah Milinovich]