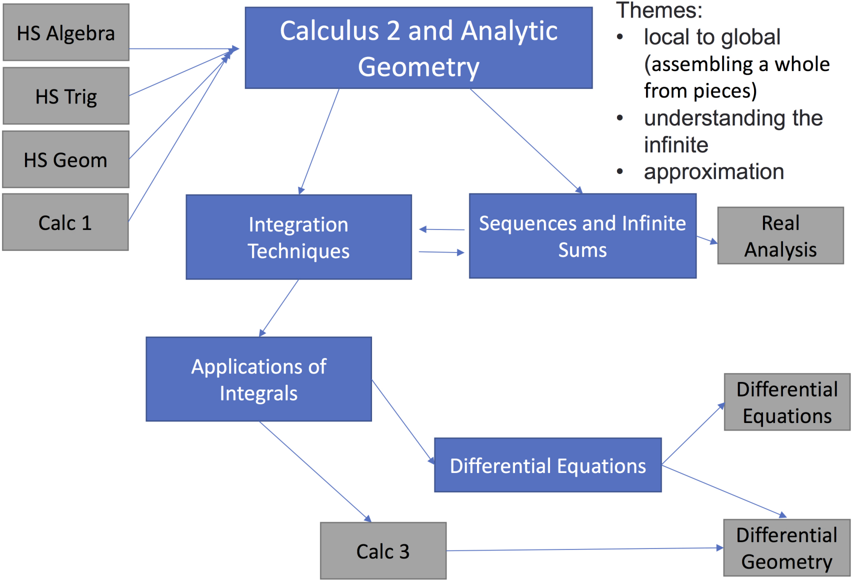
Test 1 Material

- Algebra (and lots of it!)
- Right angle trigonometry
- Limits
- Calc I derivatives and integrals, including FTC & splitting numerator
- 7.1 Substitution (Undoing the Chain Rule)
- 7.2 Parts (Undoing the Product Rule)
- 7.4 Partial Fractions (Quotients of Polynomials)
- 7.4 Trig Substitution (Apply Right Triangle Trig)
- 7.5 Numerical Integration (Approximations)
- 7.6 Improper Integrals (Infinity and Beyond)

1. *List the technique you would use to compute the antiderivative and set up the resulting transformed integral. You do NOT need to evaluate integrals in this section.*
2. *Evaluate the following integrals and show all work.*



Themes:

- local to global (assembling a whole from pieces)
- understanding the infinite
- approximation

7.1 Substitution (Undoing the Chain Rule)

- Try to find w so that dw is in $\int$
- Often helpful to choose w "inside" of some other function

What I want you to show me... w, dw, $\int$ with respect to w

$\int \cos^3 x \sin x \, dx$

$\int (\cos x)^3 \sin x \, dx$

$w = \cos x$

$dw = -\sin x \, dx$

$-dw = \sin x \, dx$

$-\int w^3 \, dw$

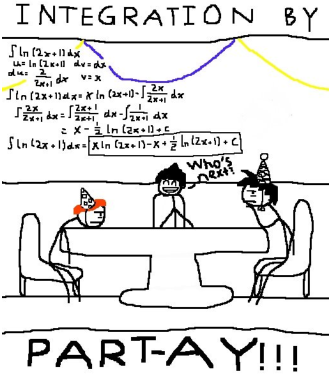
$-\frac{1}{4} w^4 + C$

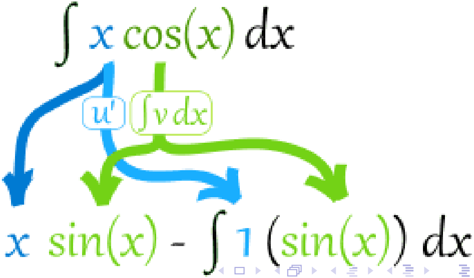
$-\frac{1}{4} \cos^4 x + C$

7.2 Parts (Undoing the Product Rule)

- Choose so $u \cdot v'$ is the function you are integrating
- If possible, choose u & v' so $\int u'v\,dx$ easier to integrate
- Might help to choose v' as the largest portion you can find an antiderivative for. Herbert Kasube: detail

What I want you to show me... $u, u', v, v', uv - \int u'v\,dx$





7.4 Partial Fractions (Quotients of Polynomials)

- Useful for us when the denominator divides up into real factors that are linear or irreducible quadratic (or repeated)
- Based on adding fractions by getting a common denominator

What I want you to show me... the expansion, and the system of equations to solve for A, B, C...

Example 1: Write $\frac{4x+1}{x^2-x-2}$ using partial fractions.

$$\frac{4x+1}{x^2-x-2} = \frac{4x+1}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2} = \frac{A(x-2)+B(x+1)}{(x+1)(x-2)}$$

$$4x+1 = A(x-2)+B(x+1)$$

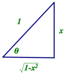
$$x=2 \quad \Rightarrow \quad 4\cdot 2+1 = A(0)+B(3) \quad \Rightarrow \quad B=3$$

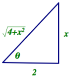
$$x=-1 \quad \Rightarrow \quad 4(-1)+1 = A(-3)+B(0) \quad \Rightarrow \quad A=1$$

Thus $\frac{4x+1}{x^2-x-2} = \frac{1}{x+1} + \frac{3}{x-2}$.

7.4 Trig Substitution (Apply Right Triangle Trig)

- If an algebraic expression looks like the Pythagorean theorem





(i.e., $\sqrt{a^2 - x^2}$ or $\sqrt{x^2 + a^2}$), then draw a triangle

Use this trig	if you see this & w-sub fails	pic	reduced radical	via algebra	via pic
$x = a\sin\theta$ $dx = a\cos\theta d\theta$	$\sqrt{a^2 - x^2}$		$a\cos\theta$	$\sqrt{a^2 - a^2\sin^2\theta}$ $=\sqrt{a^2(1 - \sin^2\theta)}$ $=\sqrt{a^2(\cos^2\theta)}$ $=\sqrt{(a\cos\theta)^2}$ $=a\cos\theta$	$\cos\theta = \frac{\sqrt{a^2 - x^2}}{a}$ mult by a
$x = a\tan\theta$ $dx = a\sec^2\theta d\theta$	$\sqrt{a^2 + x^2}$		$a\sec\theta$	$\sqrt{a^2 + a^2\tan^2\theta}$ $=\sqrt{a^2(1 + \tan^2\theta)}$ $=\sqrt{a^2(\sec^2\theta)}$ $=\sqrt{(a\sec\theta)^2}$ $=a\sec\theta$	$\sec\theta = \frac{1}{\cos\theta}$ $\sec\theta = \frac{\sqrt{a^2+x^2}}{a}$ mult by a

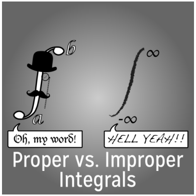
What I want you to show me... The trig sub for x, dx, triangle with sides filled in, the integral with respect to θ reduced, and (if solving) the conversions you used to get back to x.

7.6 Improper Integrals (Infinity and Beyond)

- If you see any integral with ∞ in it, or infinite discontinuities
- Express the integral as a proper one via limit (or limits) to any problem(s), like $\displaystyle\int_a^{\infty} f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$
- Integrate and evaluate the limit
- The integral converges to a finite number if the limit(s) exist, and diverges otherwise

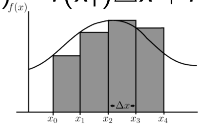
What I want you to show me... The above steps.

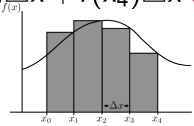
$$\int_{-2}^{0}\frac{1}{x^3}\,dx = \lim_{t\to 0^-}\int_{-2}^{t}\frac{1}{x^3}\,dx$$
$$= \lim_{t\to 0^-}\left(-\frac{1}{2x^2}\right)\Bigg|_{-2}^{t}$$
$$= \lim_{t\to 0^-}\left(-\frac{1}{2t^2}+\frac{1}{8}\right)$$
$$= -\infty$$



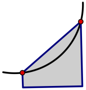
7.5 Numerical Methods

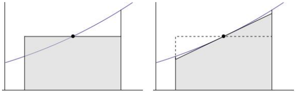
- Approximates integrals we can't evaluate directly, including discrete data
- n= number of intervals, $\triangle x = \frac{b-a}{n}$, $x_{i+1} = x_i + \triangle x$
- $Left(4) = f(x_0)\triangle x + f(x_1)\triangle x + f(x_2)\triangle x + f(x_3)\triangle x$ left endpoints
- $Right(4) = f(x_1)\triangle x + f(x_2)\triangle x + f(x_3)\triangle x + f(x_4)\triangle x$ right points

left: 

right: 

- $Trap(4) = \frac{Left(4)+Right(4)}{2}$ connect left and right points
- $Mid(4) =$
$f(\frac{x_0+x_1}{2})\triangle x + f(\frac{x_1+x_2}{2})\triangle x + f(\frac{x_2+x_3}{2})\triangle x + f(\frac{x_3+x_4}{2})\triangle x$ midpoints

trapezoid: 

midpoint: 

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