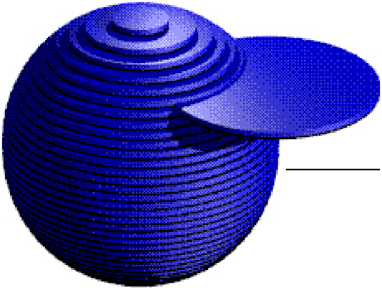
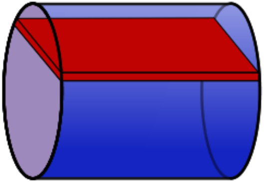
8.1 Area and Volume (Slice and Conquer)

- Area by slicing into rectangles with known length
- Volume by slicing into regions we know the area of
- Riemann sums with $\triangle x$ or $\triangle y \rightarrow \int_a^b dx$ or $\int_a^b dy$

$$\sum \pi(\tfrac{2}{5}y_i)^2 \triangle y \rightarrow \int_0^1 5(\frac{2}{5}y)^2 dy$$

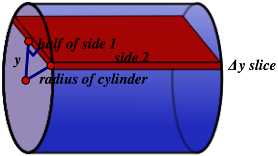
What I want you to show me... picture, slice, Riemann sum, integral





Cylinder on its Side Sliced Horizontally (Buried tank)

sliced horizontally with Δy as the height of slice
say radius=10, length of cylinder = 15, $y = 0$ at center of circle



Sketch the cylinder, Riemann slice & fill in the known lengths.

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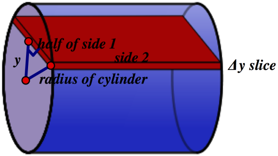

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To solve for side 1, sketch what you are looking for, and use similar triangles or Pythagorean theorem as needed.

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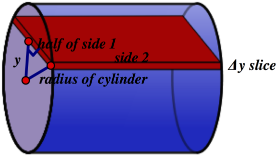

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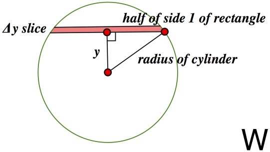

Write the Riemann sum, then the integral

Cone on its Side Sliced Vertically

say radius=5, length of cone = 12, $x = 0$ at cone point

Sketch the cone, Riemann slice & fill in known lengths.

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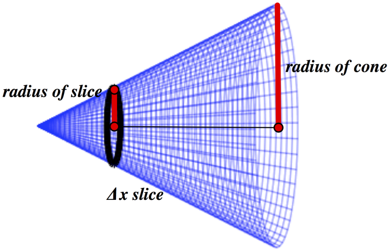

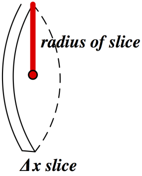

To solve for the radius of the slice, sketch what you are looking for, and use similar triangles or Pythagorean theorem as needed.

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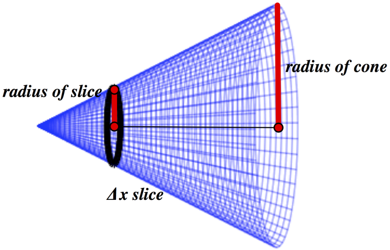
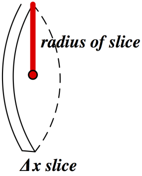
say radius=5, length of cone = 12, $x = 0$ at cone point

Sketch the cone, Riemann slice & fill in known lengths.





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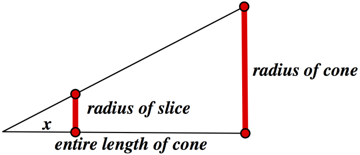
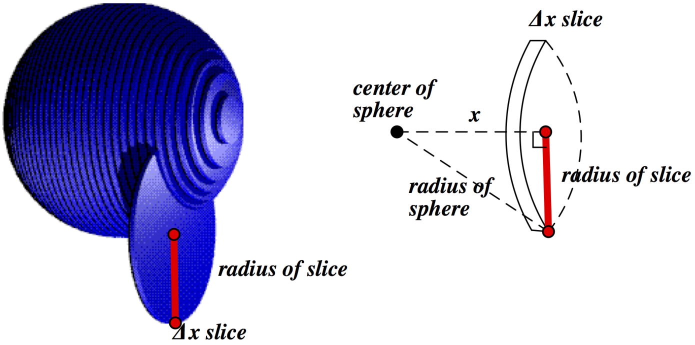


Sphere Sliced Vertically

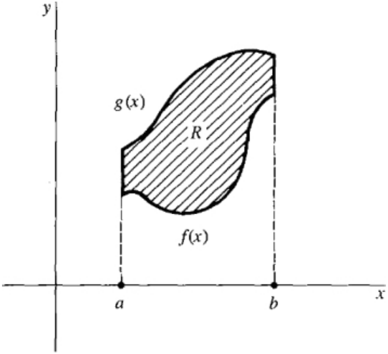
$x = 0$ at center, sphere radius r



Solve for radius of slice, which is unknown?

Clicker Question

1. The area of the region R is approximately



a) $\sum(g(x) - f(x))\triangle x$
b) $\int_a^b (f(x) - g(x))\triangle x$
c) both of the above
d) none of the above
e) no way to tell without more information

Clicker Question

2. If we slice a cylinder on its side horizontally (buried tank), then what is the approximate shape and volume of a slice

a) rectangle (length·width·height), where height is Δy or Δh

b) cylinder/disk ($\pi \cdot$ radius^2·height), where height is Δy or Δh

c) other

Clicker Question

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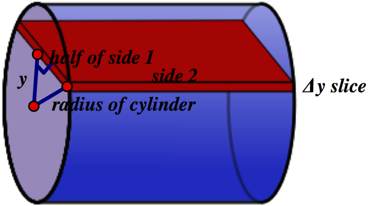
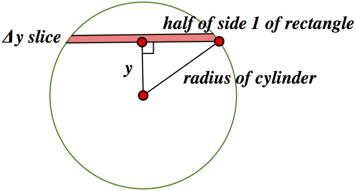
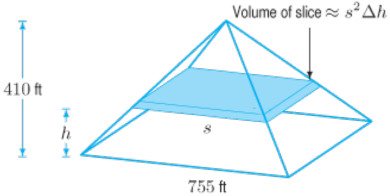




uses Pythagorean theorem

Clicker Question

3. If we slice a pyramid parallel to the base, what do we need to use to solve for the side of a slice, *s*?



a) Pythagorean theorem
b) similar triangles

Clicker Question

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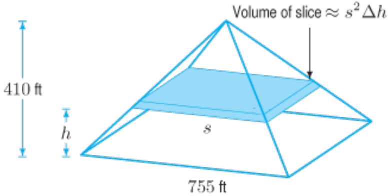

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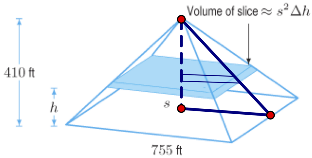
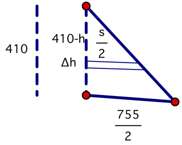


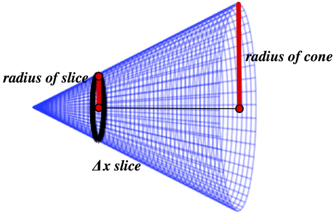


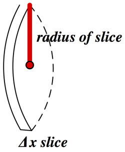
$$\frac{\frac{s}{2}}{\frac{755}{2}} = \frac{410-h}{410}, \text{ so } s = \frac{755}{410}(410-h).$$

Volume of pyramid= $\int_0^{410} \left(\frac{755}{410}(410-h)\right)^2 dh$

Clicker Question

4. If we slice a cone parallel to the base, what do we need to use to solve for the radius of the slice?

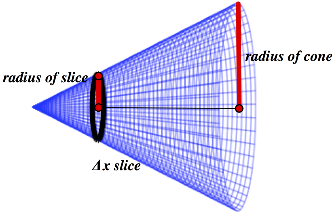


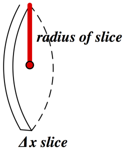


a) Pythagorean theorem
b) similar triangles
c) other

Clicker Question

4. If we slice a cone parallel to the base, what do we need to use to solve for the radius of the slice?





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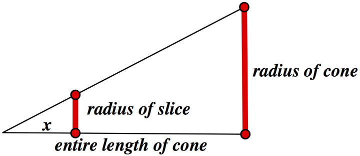

History and Applications

- Archimedes. Example: sphere + cone = cylinder
- 1821 monograph, Augustin-Louis Cauchy put forward a definition of integral that is directly based on the interpretation of area under graph of function and had limits. Before it was antideriv at endpoints.
- mathematics, physics, computer science, statistics, engineering, etc
- CT scans
- *Recent developments in volume visualization using standard graphics hardware provide an effective and interactive way to understand and interpret the data. Mainly based on 3d texture mapping, these hardware-accelerated visualization systems often use a cell-projection...*
- Volume sculpting, horizon slicing, stratal slicing