7.4 Trig Substitution (Apply Right Triangle Trig)

- If you see any algebraic expression that looks like the Pythagorean theorem (i.e., $\sqrt{a^2 - x^2}$ or $\sqrt{x^2 + a^2}$), then draw a triangle (when in doubt, appeal to geometry)

Clicker Question

1. Which pair of equations is possible?

a) $x = 4\sin(\theta)$ and $\cos(\theta) = \dfrac{\sqrt{16 - x^2}}{4}$

b) $x = 4\sin(\theta)$ and $\cos(\theta) = \dfrac{\sqrt{16 - x^2}}{x}$

c) more than one of the above

d) none of the above

Study Break:
Math Snacks

LanceAF #35 6-3-12
www.mathplane.com

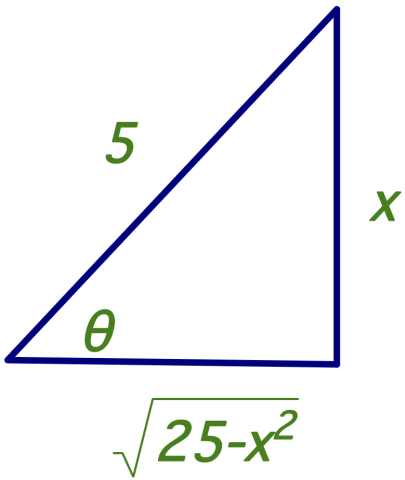
Preferable to ordinary computer cookies...

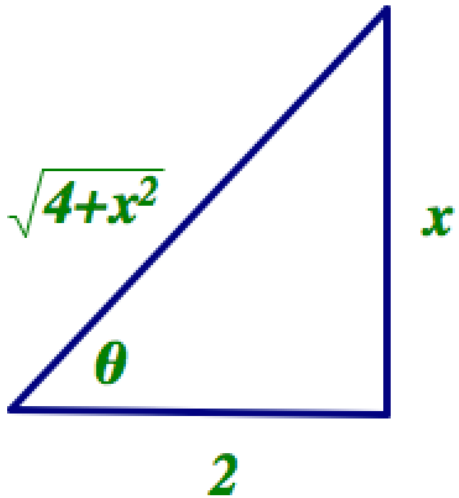
Essential part of a well-rounded, academic diet.

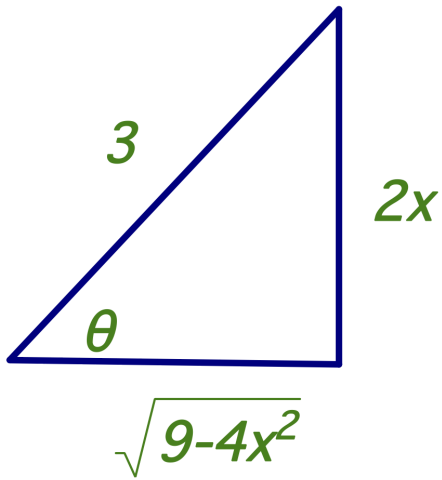
Trig Sub

Use if you see any algebraic expression that looks like the Pythagorean theorem (i.e., $\sqrt{a^2 - x^2}$ or $\sqrt{x^2 + a^2}$) and regular w-sub fails.

- Identify what trig sub to use.
- Write x and dx.
- Sketch the triangle with the sides filled in.
- Convert the integral to one with only θ.
- Simplify the radical using algebra and/or the pic.







Clicker Question

2. What is a useful method for $\int x\cos(x)\,dx$?

a) Integration by w-substitution

b) Integration by parts

c) Integration by partial fractions

d) Integration by trigonometric substitution

e) More than one of the above

Clicker Question

2. What is a useful method for $\int x\cos(x)\,dx$?

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c) Integration by partial fractions

d) Integration by trigonometric substitution

e) More than one of the above

b) product of two functions, neither the derivative of the other.

$v' =$

Clicker Question

2. What is a useful method for $\int x\cos(x)\,dx$?

a) Integration by w-substitution

b) Integration by parts

c) Integration by partial fractions

d) Integration by trigonometric substitution

e) More than one of the above

b) product of two functions, neither the derivative of the other.
$v' = \cos(x)$

Clicker Question

3. What is a useful trig substitution for integrals involving $\sqrt{x^2 + a^2}$?

a) $x = a\sin(\theta)$ since $\sqrt{x^2 + a^2} = a\cos(\theta)$

b) $x = a\tan(\theta)$ since $\sqrt{x^2 + a^2} = a\sec(\theta)$

c) more than one of the above

d) none of the above

Clicker Question

4. What is a useful method for $\int \frac{x}{1-x^4}\,dx$?

a) Integration by w-substitution

b) Integration by parts

c) Integration by partial fractions

d) Integration by trigonometric substitution

e) More than one of the above

Clicker Question

4. What is a useful method for $\int \frac{x}{1-x^4}\,dx$?

a) Integration by w-substitution

b) Integration by parts

c) Integration by partial fractions

d) Integration by trigonometric substitution

e) More than one of the above

a) $w = x^2$. Obtain $\frac{\frac{1}{2}}{1-w^2}$ which integrates to $\frac{1}{2}arctanh(x^2) + C$ and c) $1 - x^4 = (1 - x^2)(1 + x^2) = (1 - x)(1 + x)(1 + x^2)$, so $\frac{\frac{-1}{4}}{(x-1)} + \frac{\frac{-1}{4}}{(x+1)} + \frac{\frac{x}{2}}{(x^2+1)}$ both work

History and Applications

- $t = \tan(x/2)$ rational points on a circle in algebraic geometry
- finding areas of regions bounded by quadratic algebraic curves, like the area of the ellipse
- the amount of soda in a cylindrical can lying on its side given the height of the soda
- flux
- electric field around uniformly charged 1-dimensional lines in space
- object pulling another free-moving objects with rigid materials of fixed length