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weight (force in lbs) = volume of a slice $\times$62.4 lbs/ft^3
work on a slice = volume $\times$62.4 lbs/ft$^3\times$ slice displacement

Clicker Question

1. For which surfaces would we use similar triangles?

a) cone, pyramid, upside down pyramid
b) cylinder on its side like a buried tank, sphere, hemisphere
c) cylinder upright like a garbage can
d) area under the arctan $(2x)$ curve
e) other

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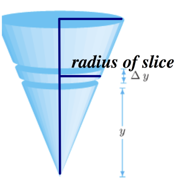

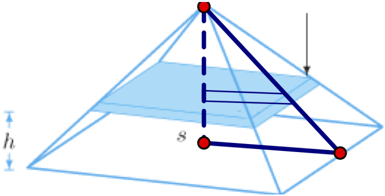

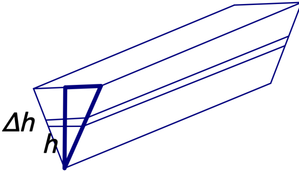

Clicker Question

2. If we have a cylindrical oil tank of radius 3 m and height 10 m standing up on its circular base above ground like a garbage can would (i.e. NOT sideways) filled to a level of 7 m, then what is the work to pump out the oil in terms of h, where h is the height from the bottom of the tank to a slice? Oil has a density of $890 kg/m^3$.

a) $\int_0^{10} 890 \times \pi 3^2 \, dh \times (7-h)$

b) $\int_0^{10} 890 \times \pi 3^2 \, dh$

c) $\int_0^{7} 890 \times \pi 3^2 \, dh \times (10-h)$

d) $\int_0^{7} 890 \times 10 \times 2\sqrt{3^2-(10-h)^2} \times dh \times h$

e) other

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d) $\int_0^{7} 890 \times 10 \times 2\sqrt{3^2 - (10-h)^2} \times dh \times h$
e) other

Work is force times the distance displaced only applies if the force is constant while it is exerted over the distance.

F d = ($890 kg/m^3 \times$ volume) $\times$ distance the slice displaced

$\int_0^{7} 890 \times \pi 3^2\, dh \times (10 - h)$

History and Applications

- Archimedes buoyant forces inherent in fluids
- Sir Isaac Newton
- Work = weight lifted through a height: 1826 French mathematician Gaspard-Gustave Coriolis
  steam engines water out of flooded ore mines