7.4 Partial Fractions and Trig Substitution and 7.6 Improper Integral in Maple

by Dr. Sarah

Maple is a computer algebra system, which was first developed in 1980 out of the at the University of Waterloo in Canada.
Maple can handle numerical, symbolic, and graphical representations. Exploring these representations and an exposure to this type of technology are course goals of Calc II.

Respond to the following questions in your notes. Read through and execute the command code by hitting return in each red line.

Example 1

Execute the following:

```
> Int(x^2/(sqrt(4-x^2)),x);
```

$$\int \frac{x^2}{\sqrt{-x^2+4}}\,dx \qquad \textbf{(1)}$$

Question 1: Which of the following methods can be used to evaluate this integral: *w*-subs, parts, partial fractions, or trigonometric substitution?

w-subs doesn't work because of the x^2 in the numerator instead of -2x, if *w* were $-x^2+4$.
It isn't a special parts (ln(x), arctan(x), arcsin(x), arccos(x) where v'=1), nor a typical parts consisting of the product of two different types of functions, as here we have algebraic and algebraic.
It isn't a partial fractions because of the square root in the denominator.
It is a trigonometric substitution, since the denominator looks like the Pythagorean theorem.
Activity: Compare your response with a neighbor.

Maple's int feature integrates the function without specifying the method it uses:

```
> int(x^2/(sqrt(4-x^2)),x);
```

$$-\frac{x\sqrt{-x^2+4}}{2}+2\arcsin\left(\frac{x}{2}\right) \qquad \textbf{(2)}$$

Please note that it should be +c at the end, but Maple leaves that off!

Question 2: First compare your response to Question 1 with Maple's output to see if you wish to modify your response or not. Even though Maple doesn't list the integration method, the output can still give you clues about what method(s) it was. Next:

If it is a *w*-subs, write down what *w* is.

If it is parts, write down *u* and *v'*.

If it is partial fractions, write the fraction decomposition, like $\frac{A}{x+5}+\ldots$

If it is trigonometric substitution, write down x and draw the triangle and label its sides.

Here x=2 sin(theta). The picture would show a right triangle with x the side opposite the angle theta, $\sqrt{4-x^2}$ as the adjacent side, and 2 as the hypotenuse. The adjacent side is also 2 cos(theta).

Example 2

Execute the following:

```
> Int(2*x/(x^2-x-6),x);
```

$$\int \frac{2x}{x^2-x-6}\,dx \tag{3}$$

Question 3: Which of the following methods can be used to evaluate this integral: w-subs, parts, partial fractions, or trigonometric substitution?

w-subs doesn't work because of the 2 x in the numerator instead of 2x-1, if w were x^2-x-6.
It isn't a special parts (ln(x), arctan(x), arcsin(x), arccos(x) where v'=1), nor a typical parts consisting of the product of two different types of functions, as here we have algebraic and algebraic.
It isn't a trigonometric substitution, since nothing here looks like the Pythagorean theorem.
It is a partial fractions integral.

Activity: Compare your response with a neighbor.

Maple's int feature integrates the function without specifying the method it uses:

```
> int(2*x/(x^2-x-6),x);
```

$$\frac{6\ln(x-3)}{5}+\frac{4\ln(x+2)}{5} \tag{4}$$

Please note that it should be +c at the end, but Maple leaves that off!

Question 4: Compare your response to Question 3 with Maple's output to see if you wish to modify your response or not. Even though Maple doesn't list the integration method, the output can still give you clues about what method(s) it was. Next:

If it is a w-subs, write down what w is.

If it is parts, write down u and v'.

If it is partial fractions, write the fraction decomposition, like $\frac{A}{x+5}+\ldots$

If it is trigonometric substitution, write down x and draw the triangle and label its sides.

$$\frac{A}{x-3}+\frac{B}{x+2}$$

Example 3

Execute the following, which is the same integral from Example 1 but with endpoints:

```
> Int(x^2/(sqrt(4-x^2)),x=0..2);
```

$$\int_0^2 \frac{x^2}{\sqrt{-x^2+4}}\,dx \tag{5}$$

Question 5: Is the integral a proper or improper integral? If it is an improper integral, the write out the limit integral set up.

The integral is improper because 2 makes the denominator 0.

$$\text{limit } b \to 2^- \int_0^b \frac{x^2}{\sqrt{-x^2+4}}\,dx$$

Activity: Maple can also evaluate such integrals:

```
> int(x^2/(sqrt(4-x^2)),x=0..2);
```

$$\pi \tag{6}$$

Compare this response with your limit statement as well as Maple's output just before Question 2 in Example 1 in order to see where it comes from.

$$\text{limit } b \to 2^- \left(-\frac{x\sqrt{-x^2+4}}{2} + 2\arcsin\left(\frac{x}{2}\right) \right) \Bigg|_0^b$$

$$= \text{limit } b \to 2^- \left(-\frac{b\sqrt{-b^2+4}}{2} + 2\arcsin\left(\frac{b}{2}\right) \right) - \left(-\frac{0\sqrt{-0^2+4}}{2} + 2\arcsin\left(\frac{0}{2}\right) \right)$$

$$= 0 + \frac{2\pi}{2} - (0+0) = \pi$$

as $\arcsin\left(\frac{b}{2}\right)$ goes to $\arcsin\left(\frac{2}{2}\right) = \arcsin(1) = \frac{\pi}{2}$ and $\arcsin(0) = 0$

Example 4

Execute the following, which is the same integral from Example 2 but with endpoints:

```
> Int(2*x/(x^2-x-6),x=3..4);
```

(7)

$$\int_3^4 \frac{2x}{x^2 - x - 6}\, dx \qquad \textbf{(7)}$$

Question 6: Is the integral a proper or improper integral? If it is an improper integral, the write out the limit integral set up.

$$\lim_{a \to 3^+} \int_a^4 \frac{2x}{x^2 - x - 6}\, dx$$

Activity: Maple can also evaluate such integrals:

```
> int(2*x/(x^2-x-6),x=3..4);
```

$$\infty \qquad \textbf{(8)}$$

Compare this response with your limit statement as well as Maple's output just before Question 4 in Example 2 in order to see where it comes from.

$$\lim_{a \to 3^+} \left. \frac{6\ln(x-3)}{5} + \frac{4\ln(x+2)}{5} \right|_3^4$$

$$= \lim_{a \to 3^+} \frac{6\ln(4-3)}{5} + \frac{4\ln(4+2)}{5} - \left(\frac{6\ln(a-3)}{5} + \frac{4\ln(a+2)}{5} \right)$$

The third term is undefined as it tends to ln(0) as a approaches 3 so the integral diverges.