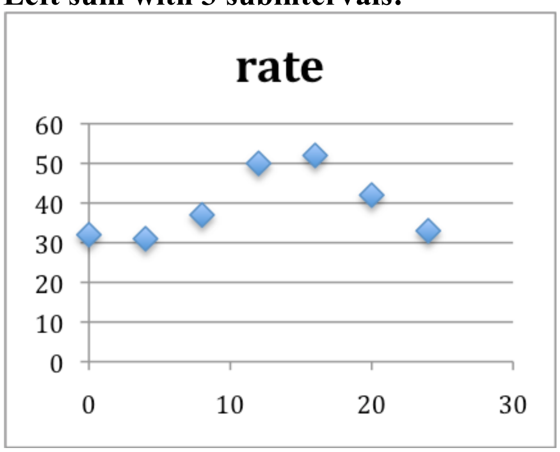
7.5 Numerical Integration

Ex 1: Pipelines tend to be hundreds of miles long. This enormous length increases their tendency to leak and complicates the task of locating leaks. The rate at which oil flows through the pipeline depends on (among other things) the viscosity (thickness) of the oil and this viscosity depends on the temperature of the oil. The higher the temperature, the lower the viscosity and the faster the oil flows; and the lower the temperature, the higher the viscosity and the slower the oil flows. To facilitate detecting leaks, an oil company makes two types of measurements. First, they measure the amount of oil pumped into the tanks at the refinery. Second, meters are installed along the pipeline to measure the rate at which oil is flowing at various strategic locations. Every 4 hours over a 24-hour period, a technician observes the rate in barrels per hour at which oil is flowing through the pipeline at a specific meter.

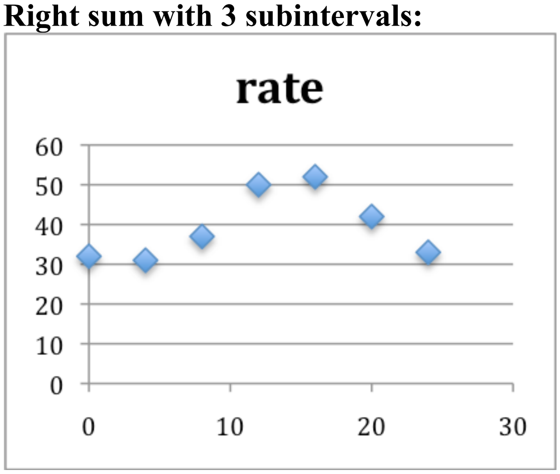
Time (hrs)	0	4	8	12	16	20	24
Rate (barrels/hr)	32	31	37	50	52	42	33

Sketch the numerical method and then calculate an estimate of the total number of barrels pumped during the 24-hour periods for each of the following:

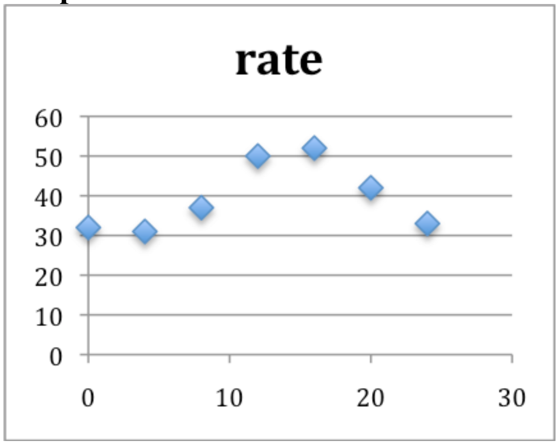
Left sum with 3 subintervals:



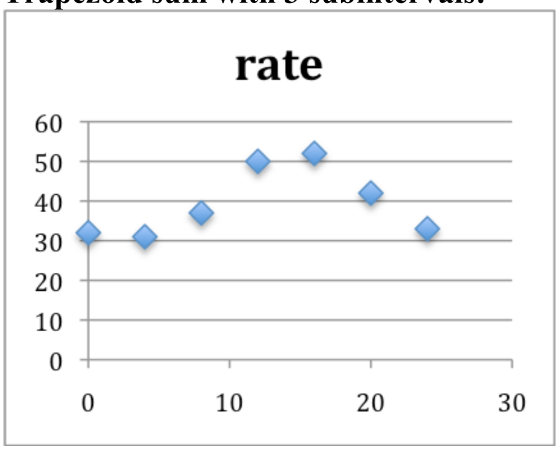
Right sum with 3 subintervals:



Midpoint sum with 3 subintervals:



Trapezoid sum with 3 subintervals:



<u>**Ex 2**</u>**:** An artificial lake is created as a reservoir for agricultural irrigation. The rate at which the water flows out of the lake is regulated by a state agency. The table below gives the rate in hundreds of cubic feet per day at which the water is released every two days for 10 days.

Time (days)	0	2	4	6	8	10	12
Hundreds of cubic feet per day	120	110	100	80	50	20	15

Estimate the total amount of water that is released in that period using

a. A trapezoid sum with 6 subintervals.

b. A midpoint sum with 3 subintervals.

c. Is a midpoint sum with 6 intervals possible?

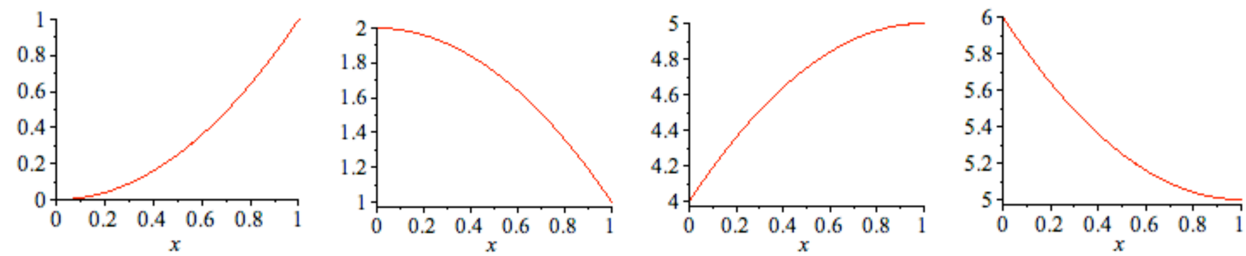
<u>Ex 3</u>: Estimate $\int_0^1 e^{-x^2}\,dx$ using

a. A left sum with 2 subintervals.

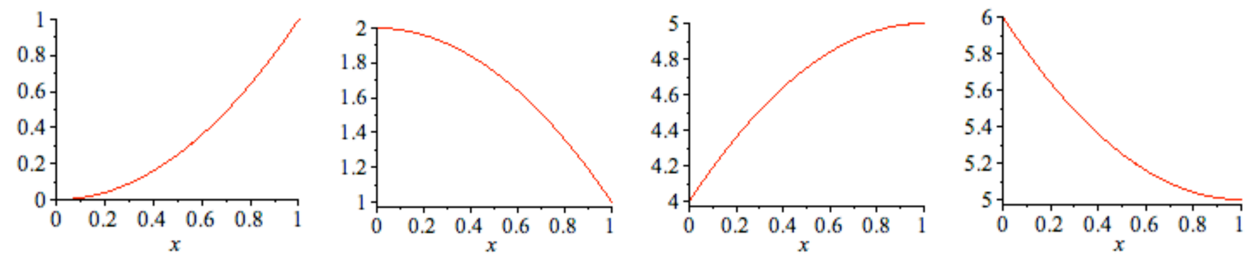
b. A right sum with 4 subintervals.

Below are some graphs. For each of them, draw in the **left, right, and trapezoid** approximation with **1 subinterval** over the interval [0,1].

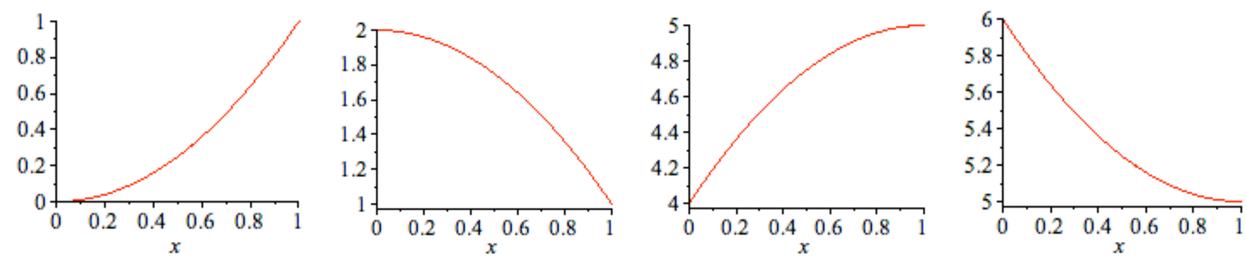
Left(1):



Right(1):



Trap(1):



For each picture, determine if the **left, right, and trapezoid approximations** are **overestimates** or **underestimates.** Write **OVER or UNDER** in the blocks below.

	Picture 1	**Picture 2**	**Picture 3**	**Picture 4**
Left Sum				
Right Sum				
Trapezoid Sum				

Determine what property of the function causes each of the approximations to be **overestimates** or **underestimates.**

Summarize your finding by completing the following statements:

The **Left Sum** is an **underestimate** of $\int_a^b f(x)\,dx$ if the function $f(x)$ is ____________________

and is an **overestimate** if $f(x)$ is __________________________________.

The **Right Sum** is an **underestimate** of $\int_a^b f(x)\,dx$ if the function $f(x)$ is __________________

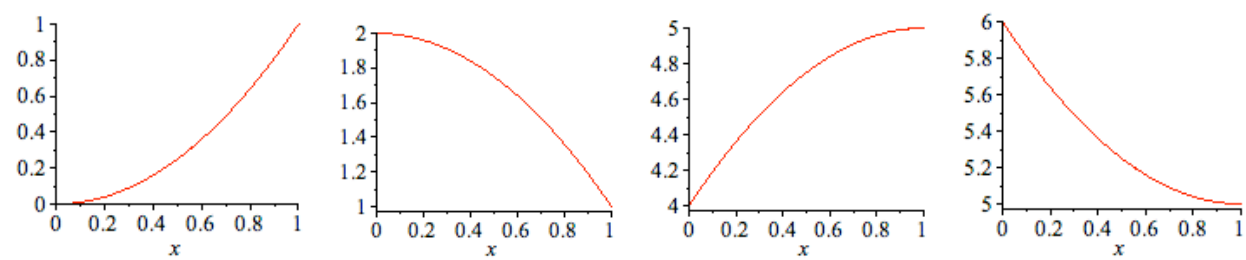
and is an **overestimate** if $f(x)$ is __________________________________.

The **Trapezoid Sum** is an **underestimate** of $\int_a^b f(x)\,dx$ if the function $f(x)$ is _______________

and is an **overestimate** if $f(x)$ is __________________________________.

In the pictures below, draw in the tangent line at the midpoint and use that trapezoid (which gives the same area as the midpoint rectangle) to determine if the midpoint is an **overestimate** or **underestimate** for the area under each graph. Then fill in the following:

Mid(1):



The **Midpoint Sum** is an **underestimate** of $\int_a^b f(x)\,dx$ if the function $f(x)$ is _______________

and is an **overestimate** if $f(x)$ is __________________________________.