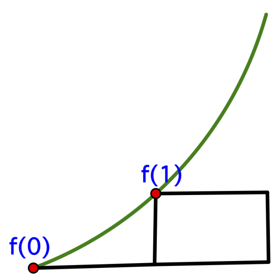
7.5 Group Work Target Practice

Directions: For each problem below, set up, and fill in with numbers, but DO NOT evaluate and DO NOT simplify, like $\ln|5-3|$.

1. $\int_0^2 x^2\, e^{x^2}\, dx$ using LEFT(2).

   First we compute the width of each of the rectangles: n is 2 because of LEFT(2), and we obtain a and b from the region we want the integral of: $\Delta x = \frac{b-a}{n} = \frac{2-0}{2} = 1$. This makes sense because we want 2 subintervals in going from 0 to 2, so the first goes from 0 to 1, and the second from 1 to 2.

   Sketching a number line or graph may help, and we will take the left endpoint of each of the subintervals to plug in to obtain the height of the rectangles.

   Numerically LEFT(2) means we add the base $\times$ height of each rectangle, where the base is $\Delta x = 1$

   $1 \times f(0)\ +\ 1 \times f(1)$

   $1 \times 0^2\, e^{0^2}\ +\ 1 \times 1^2\, e^{1^2}$

2. $\int_0^{13} f(x)\, dx$ as pictured in the table, using TRAP(1).

x	0	3	7	13
f(x)	10	15	18	21

   TRAP(1) is the average of LEFT(1) and RIGHT(1).

   First $\Delta x = \frac{13-0}{1} = 13$

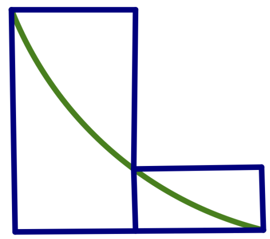
   LEFT(1) is the base of 13 $\times$ height using the left endpoint of 0: $13 \times f(0) = 13 \times 10$

   RIGHT(1) is the base of 13 $\times$ height using the right endpoint of 13: $13 \times f(13) = 13 \times 21$

   TRAP(1) averages these: $\frac{13 \times 10 + 13 \times 21}{2}$

3. Sketch a graph of x^2 from $x = -1$ to $x = 0$ as well as a sketch of LEFT(2).

   Between -1 and 0 x^2 is decreasing and concave up. LEFT(2) breaks the interval up into two regions of equal width (each of width $\frac{1}{2}$ here because of $\frac{b-a}{n} = \frac{0-(-1)}{2}$), and we use the left function value for each interval to obtain the height of each of the rectangles:

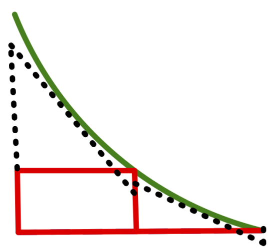


4. If a function is decreasing and concave up, create a ranking, from smallest to largest:

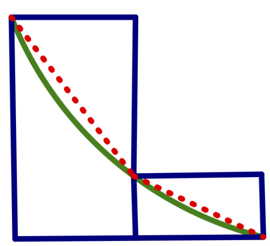
   LEFT(n), RIGHT(n), the integral of the function, MID(n), TRAP(n)

   You can create sketches just like the one above for the other images. We'll see that left is an overestimate, right is an underestimate, because the function is decreasing. We'll see that mid is an underestimate and trap is an overestimate because the graph is concave up.

   Also, mid, using the slope of the tangent line at the midpoint, is an underestimate but closer to the value of the integral than right is—sketch the graphs to see this. Here is right and mid, both underestimates here, with mid closer to to the value of the integral—the area under the curve:

   Here is left and trap, both overestimates here, with trap are closer to the value of the integral—the area under the curve.

   RIGHT(n) $<$ MID(n) $<$the integral of the function $<$ TRAP(n) $<$ LEFT(n)