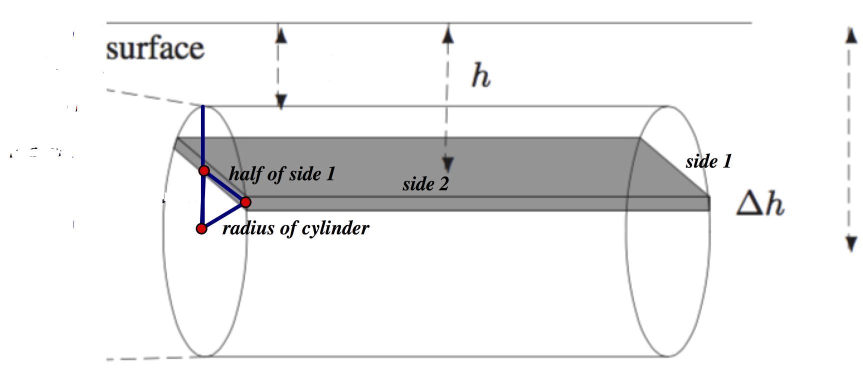
Area and Volume by Slicing in 8.1 and 8.2
Group Work Target Practice

1. Look at the region between $\sin(x)$ and $\cos(x)$ from 0 to $\frac{\pi}{4}$. Find the area via slicing:
   (a) Sketch a graph of the functions to find the enclosed region
   (b) Sketch a picture of a Riemann slice on your graph.
   (c) Base of the rectangle? Circle: Δx or Δy
   (d) Which function is larger in that variable (top for x, right for y)?
   (e) What is the height of the rectangle (top-bottom or right-left)?
   (f) What is the Riemann sum approximation? $\sum$ height $\cdot$ base $= \sum$
   (g) What are the limits of the integral a and b (algebra finds the intersection points, if not already given)?
   (h) Write the integral. Set up but do not solve.
   (i) How could we integrate this? Identify but do not evaluate the integral.

2. For the region from # 1, that is the region between $\sin(x)$ and $\cos(x)$ from 0 to $\frac{\pi}{4}$, find the volume by revolving the same region about the $x-$axis
   (a) Slice perpendicular to this axis of revolution. Sketch a picture of a Riemann slice.
   (b) Which is the infinitesimal part of the slice? Circle: Δx or Δy
   (c) Is the slice a solid cylindrical region or an annular/washer region?
   If it is a solid region, what is r in terms of the integration variable?
   If it is an annular region, what is r_{outer}? What is r_{inner} in terms of this variable?
   (d) Set up the integral that gives the volume and identify geometric/physical components.
   Common forms: $\int_a^b \pi r^2 dx$ or $\int_a^b \pi(r_{outer}^2 - r_{inner}^2)dx = \int_a^b \pi r_{outer}^2 dx - \int_a^b \pi r_{inner}^2 dx$
   (e) How could we integrate this? Identify but do not evaluate the integral.

3. A gas station stores its gasoline in a tank under the ground. The tank is a cylinder lying horizontally on its side. The radius of the cylinder is 4 feet, its length is 12 feet, and its top is 10 feet under the ground. Sometimes it makes sense to define the slicing variable y from the center of the tank to a slice, like we have seen in previous examples. Here it makes sense to define the slicing variable h from the ground to a slice, i.e. the distance underground. Find the volume of the cylinder in terms of h via slicing as per below.



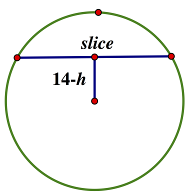
   (a) Sketch the object you want to find the volume of (it is above, but your own sketch will help you internalize it)

(b) Sketch a picture of a Riemann slice on your graph
(c) What shape is the slice? Circle: box (length·width·height) or cylinder/disk (π·radius^2·height)
(d) Infinitesimal part of the slice? Circle: Δx or Δy or Δh or Δr
(e) To solve for any lengths you need, sketch a diagram and show work.

h is defined as the distance from ground to a slice so label this on your diagram. Also label 10 feet from ground to the top of the tank, and a 4 foot radius from the top of the tank to the center of the circle.

Next, be sure that you can visualize why 14 is from ground to the center of the circle and why $14 - h$ is the distance from the center of circle to a slice.

(f) Next make a right triangle in this diagram and circle any you use to solve for the slice width: Pythagorean theorem or similar triangles



(g) What is the Riemann sum approximation? $\sum$
(h) What is a and b?
(i) Write the integral?
(j) What method could we use to evaluate? Identify the method but do not solve.

4. For $\arctan(2x)$ from 0 to 1

(a) Sketch a graph over the interval.

(b) Set up the arc length of the curve.

(c) Sketch the region R that gives the area under the curve. Set up the integral that gives the area under the curve and identify geometric/physical components.

(d) Set up the integral that gives the volume by revolving the region R about the $x-$axis and identify geometric/physical components

(e) Set up the integral that gives the volume by revolving the region R about the $y-$axis and identify geometric/physical components

(f) Which of the above, if any, can we successfully use w-subs, parts, partial fractions, trig subs, or improper on? Identify but do not evaluate the integrals.

5. Find the volume of the cone standing on its tip via slicing horizontally if the cone has a height of 5 and radius of 2 by following the instructions below.

(a) Sketch the object you want to find the volume of
(b) Sketch a picture of a Riemann slice on your graph
(c) What shape is the slice? Circle: box (length·width·height) or cylinder/disk (π·radius^2·height)
(d) Infinitesimal part of the slice? Circle: Δx or Δy
(e) To solve for any lengths you need, sketch a diagram and show work.
(f) Circle any we used: Pythagorean theorem or similar triangles
(g) What is the Riemann sum approximation? $\sum$
(h) What is a and b?
(i) Write the integral? Set up but do not solve.