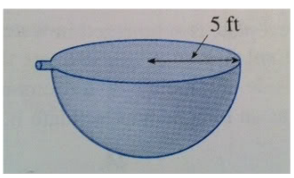
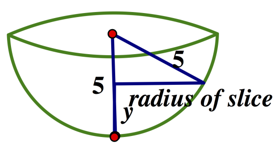
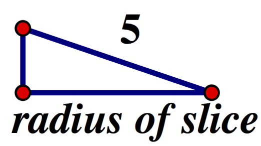
8.4 Density and 8.5 Work
Group Work Target Practice

1. The density of oil in a circular oil slick on the surface of the ocean at a distance meters from the center of the slick is given by $\delta(r) = \frac{50}{1+r} kg/m^2$. The slick extends from $r = 0$ to $r = 10,000m$. There are many interesting applications, including within what distance half the oil of the slick is contained.
   (a) Slice perpendicular to the density variable and sketch a picture of the resulting slice that has approximately constant density:
   (b) To find the area of the slice, which of the following should we use?
   length $\times$ width $2\pi r \Delta r \approx \pi(r^2_{\text{outer}} - r^2_{\text{inner}})$ πr^2 $(g(x) - f(x))\Delta x$
   (c) Find a Riemann sum approximating the total mass of oil in the slick.
   (d) Find the exact value of the mass of oil in the slick by turning your sum into an integral and evaluating it—look at the r portion and rewrite the numerator as r=(1+r) -1, so that you can break up the integral into two components before you integrate.

2. A hemispherical tank of radius $5ft$ is filled with gasoline, which weighs $42lb/ft^3$. How much work is required to pump all the fluid to the top rim of the tank? Let y be the height from the bottom of the sphere to a slice.



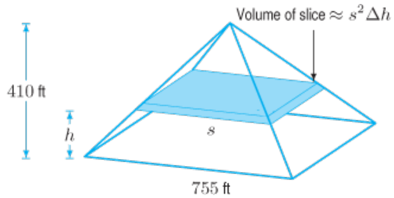




   (a) What is the shape of the slice? rectangular cylinder/disk annulus/washer
   (b) What is the unlabeled height on the picture on the right (as a function of y)?
   (c) What do we need to solve for the radius of the slice?
   Pythagorean theorem similar triangles neither
   (d) What is the volume of the slice?
   (e) What is the work required for that slice?
   F d = (volume $\times 42lb/ft^3$)$\times$ distance the slice must be displaced=
   (f) Set up the integral for the total work required.

3. Mixed practice for a cone of radius 2 ft and height 5 ft standing on its tip
   (a) What is the volume of one slice of the cone via slicing horizontally. Show reasoning and a picture.
   (b) Find the volume of the entire cone via slicing horizontally. Show reasoning and pics.
   (c) If the density $\delta(y)$ of the cone varies with it's height y, set up the integral that represents the total mass. Show reasoning.
   (d) If the cone is partially filled with fresh water that weighs 62.5 lbs/ft^3 to a height of 4 ft, what is the total work required to pump the water out over the top? Show reasoning and pics and set up the integral.

4. It is reported that the Great Pyramid of Egypt was built in 20 years. If the stone making up the pyramid has a density of 200 pounds per cubic foot, find the total amount of work done in building the pyramid. The pyramid is 410 feet high and has a square base 755 feet by 755 feet. Assume that the stones were originally located at the approximate height of the construction site and were square slices. Imagine the pyramid was built in layers



   (a) What do we need to solve for $\frac{s}{2}$, with s as in the picture?

   Pythagorean theorem   similar triangles   neither

   (b) Set up the integral for the total work required and show reasoning.

5. Medical Case Study: Testing for Kidney Disease

   Patients with kidney disease often have protein in their urine. While small amounts of protein are not very worrisome, more than 1 gram of protein excreted in 24 hours warrants active treatment. The most accurate method for measuring urine protein is to have the patient collect all his or her urine in a container for a full 24 hour period. The total mass of protein can then be found by measuring the volume and protein concentration of the urine. However, this process is not as straightforward as it sounds. Since the urine is collected intermittently throughout the 24 hour period, the first urine voided sits in the container longer than the last urine voided. During this time, the proteins slowly fall to the bottom of the container. Thus, at the end of a 24 hour collection period, there is a higher concentration of protein on the bottom of the container than at the top. One could try to mix the urine so that the protein concentration is more uniform, but this forms bubbles that trap the protein, leading to an underestimate of the total amount excreted. A better way to determine the total protein is to measure the concentration at the top and at the bottom, and then calculate the total protein.

   (a) Suppose a patient voids 2 litres (2000 ml) of urine in 24 hours and collects it in a cylindrical container of diameter 10 cm (note that $1cm^3 = 1ml$). A technician determines that the protein concentration at the top is .14 mg/ml, and at the bottom is .96 mg/ml. Assume that the concentration of protein varies linearly from the top to the bottom. Find a formula for the protein concentration in mg/ml, as a function of y, the distance in centimeters from the base of the cylinder.

   (b) Determine the quantity of protein in a slice of the cylinder.

   (c) Write an integral that gives the total quantity of protein in the urine sample. Does this patient require active treatment?

6. Mixed practice for a cylinder of radius 4 ft and length 12 ft laying sideways

   (a) If the cylinder is buried 10 feet under ground, with the height h measured from ground level to a slice, and it is completely filled with water, which weighs 64 lbs/ft^3 (more than fresh water which is 62.5 lbs/ft^3), what is the work required to pump the salinated water above ground? Show reasoning and pics and set up the integral.

   (b) If the density $\delta(h)$ of material in a cylinder varies with it's height h, set up the integral that represents the total mass. Show reasoning.