8.4 Density and 8.5 Work Group Work Target Practice

1. A hemispherical tank of radius $7ft$ is filled with gasoline, which weighs $42lb/ft^3$. How much work is required to pump all the fluid to the top rim of the tank? Let y be the height from the south pole of the hemisphere to a slice. Identify physical and geometric components and show work and reasoning as you set up (but do not evaluate) for the total work required in terms of y.

2. Mixed practice for a cone of radius 2 ft and height 5 ft standing on its tip, with y from the cone point.
   (a) What is the volume of one slice of the cone in terms of y via slicing horizontally. Show reasoning.
   (b) Set up the volume of the entire cone in terms of y. Identify physical and geometric components.
   (c) If the density $\delta(y)$ of the cone varies with it's height y, slice perpendicular to the density variable and sketch a picture of the resulting slice that has approximately constant density. Then set up the total mass, leaving $\delta(y)$ general. Identify geometric and physical components.
   (d) If the cone is partially filled with fresh water that weighs 62.5 lbs/ft^3 to a height of 4 ft, what is the total work required to pump the water out over the top? Identify geometric and physical components.

3. It is reported that the Great Pyramid of Egypt was built in 20 years. If the stone making up the pyramid has a density of 200 pounds per cubic foot, find the total amount of work done in building the pyramid. The pyramid is 410 feet high and has a square base 755 feet by 755 feet. Assume that the stones were originally located at the approximate height of the construction site and were square slices, and that h is measured from the ground to a slice. Imagine the pyramid was built in layers by lifting slices from the ground on up. Set up the integral for the total work required in terms of h and show reasoning. Identify geometric and physical components.

4. Mixed practice for a cylinder of radius 4 ft and length 12 ft laying sideways
   (a) If a cylinder is buried 10 feet under ground, with the height h measured from ground level to a slice, and it is completely filled with water, which weighs 64 lbs/ft^3 (more than fresh water which is 62.5 lbs/ft^3), what is the work required to pump the salinated water above ground? Show reasoning and pics and identify geometric and physical components.
   (b) If the density $\delta(h)$ of material in a cylinder varies with it's height h, set up the integral that represents the total mass. Identify geometric and physical components.

5. Medical Case Study: Testing for Kidney Disease

   Patients with kidney disease often have protein in their urine. While small amounts of protein are not very worrisome, more than 1 gram of protein excreted in 24 hours warrants active treatment. Since the urine is collected intermittently throughout the 24 hour period, the first urine voided sits in the container longer than the last urine voided. During this time, the proteins slowly fall to the bottom of the container. Thus, at the end of a 24 hour collection period, there is a higher concentration of protein on the bottom of the container than at the top.
   (a) Suppose a patient voids 2 litres (2000 ml) of urine in 24 hours and collects it in a cylindrical container of diameter 10 cm (note that $1cm^3 = 1ml$). A technician determines that the protein concentration at the top is .14 mg/ml, and at the bottom is .96 mg/ml. Assume that the concentration of protein varies linearly from the top to the bottom. Find a formula for the protein concentration in mg/ml, as a function of y, the distance in centimeters from the base of the cylinder.
   (b) Determine the quantity of protein in a slice of the cylinder.
   (c) Write an integral that gives the total quantity of protein in the urine sample. Does this patient require active treatment?