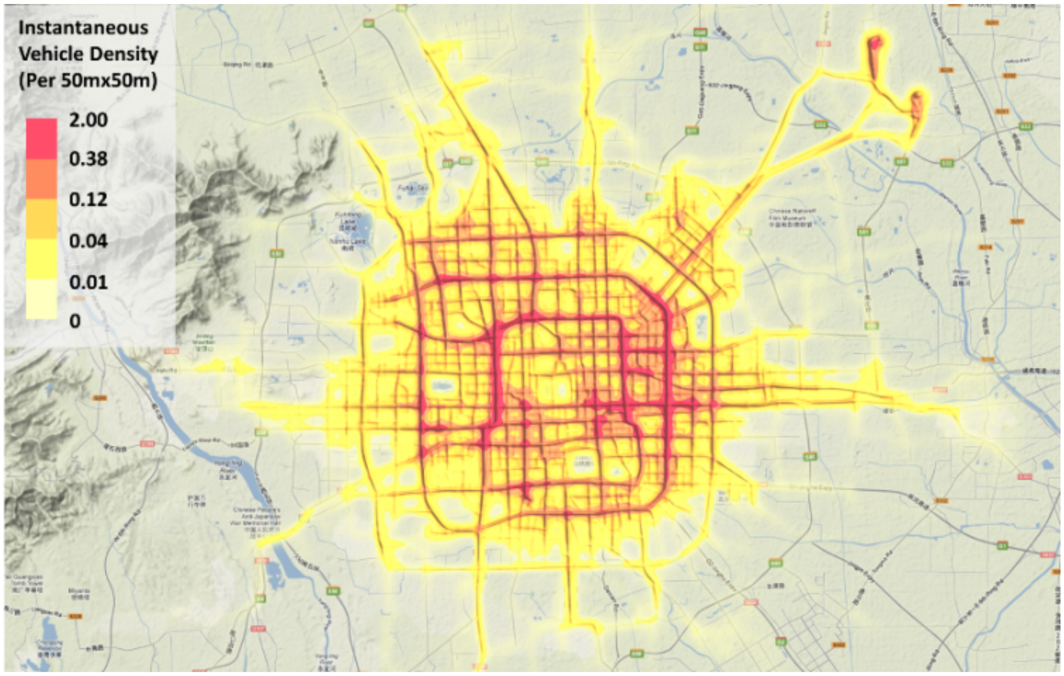
8.4 Density Applications of 8.1 and 8.2

The mass of a substance or a population count is often computed as a density times a length, area, or volume. However, this only applies if the density is constant over the region, like you would have learned about starting in middle grades science classes. There are many scenarios where the density is not a constant. Riemann sum approximations or integrals apply when we vary the density over a region, like traffic density:



by Zuchao Wang and Xiaoru Yuan, who note that: "Traffic density rendering is important, because it gives an intuitive overview of massive trajectories. It is also related to the study of traffic jams, hot spots and people's behaviors. We use density map algorithm to generate pictures of traffic density in Beijing, with real taxi GPS data." (http://vis.pku.edu.cn/trajectoryvis/en/densitymap.html)

In Calculus II, we will vary the density in one direction (Calculus III would handle more complex situations). The idea is to slice so that the density is approximately constant on a slice.

1. As a simplified example of traffic density, suppose that the function $\delta(x) = 200 + 100e^{-.1x}$ models the density of traffic on a straight road, where $\delta(x)$ is measured in cars per mile and x is the number of miles east of a major interchange.

    (a) What are the units on $\delta(x)\Delta x$, where Δx is a small slice of the road?

    (b) We'll add up the different densities on each slice and form the Riemann sum
    $\sum \delta(x) \cdot$ part of the road where density is approximately contstant $= \sum \delta(x)\Delta x$
    $= \sum 200 + 100e^{-.1x} \quad \Delta x$. Then take the limit for the definite integral: $\int_0^2 200 + 100e^{-.1x} \quad dx$
    What integration method from 1120 applies here?
    Evaluate the integral.

    (c) What is the meaning of the value you found in this real-life context?

2. Picture a cone that has a radius of 4 m and a length of 5 m lying horizontally as a solid of revolution.

   (a) Sketch and label with known lengths.

   (b) Show reasoning/work and additional sketches to solve for any needed lengths expressed in terms of the distance from the cone point, x.

   (c) Use this to set up a definite integral whose value is the volume of the cone.

   (d) Next, suppose that the cone has uniform density of 800 kg/m^3. What is the mass of the solid cone, as an integral?

   (e) Now suppose that the cone's density is not uniform, but rather that the cone is most dense towards the cone point. In particular, assume that the density of the cone is uniform across cross sections parallel to its base, but that in each such cross section that is a distance x units from the origin, the density of the cross section is given by the function $\delta(x) = 400 + \dfrac{200}{x^2}$, measured in kg/m^3. First think about the mass of a given slice of the cone x units away from the cone point; remember that in such a slice, the density will be essentially constant, so multiply $\delta\cdot$ volume of a slice.

   (f) Now add up all the slices in a Riemann sum for the mass.

   (g) Then set up a definite integral whose value is the mass of this cone of non-uniform density.

   (h) What integration methods from 1120 apply here?

3. Group Work:

   Appalachian's General Education Program prepares students to employ various modes of communication. Successful communicators interact effectively with people of both similar and different experiences and values.

   (a) Swap papers with a neighbor or two. Look through your neighbor's paper and circle anything that differed from your paper or that you have a question on.

   (b) Next, hand back the paper and discuss.

Portions of Questions 1 and 2 were adapted from *Active Calculus*.