9.4 Series
Group Work Target Practice

1. Go through each of our series tests on the Series Theorems sheet in 9.2–9.4, as well as the conditions/assumptions for the tests, and write down whether each can be applied or not, and if so, whether the test determines convergence for $\sum_{n=1}^{\infty} \frac{n^2}{n^3+6}$

   (a) Is this a geometric series? yes no
   Briefly, why or why not?

   (b) Can we apply the Terms not Going to 0 $\sum a_n$, $a_n \not\to 0$? yes no
   Briefly, why or why not?

   (c) Are the terms decreasing and positive eventually, and if so is this an integral we can do? yes no
   Briefly, why or why not?

   (d) Are the terms positive, and if so, can we limit compare the series with another to obtain $0 < \lim_{n->\infty} \frac{a_n}{b_n} < \infty$? yes no
   Briefly, why or why not?

   (e) Does the ratio test naturally apply (factorials ! or exponentials like 2^n that aren't already geometric) and if so does that give us $L < 1$ or $L > 1$? yes no
   Briefly, why or why not?

   (f) Is this an alternating series? yes no
   If so, can we make use of the alternating series test yes no
   Briefly, why or why not?

   Select one test to apply and fully document, including assumptions:

For sequences EXPLAIN or SHOW WORK documenting why your answer is correct:

a) write "sequence." does it converge or diverge, and why

b) what is the limit if it converges?

c) show work for L'Hôpital's Rule if it applies.

For series EXPLAIN or SHOW WORK documenting why your answer is correct:

a) (LG 3) choose a series test we can successfully use on it from the Series Theorems sheet and write the name of the test

b) fully document why the series test works, including any assumptions

c) specify whether the series converges or diverges, and why

1. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n-1}\frac{n}{n+5}$

2. $\displaystyle\sum_{n=0}^{\infty}\frac{n}{n!}$

3. $\displaystyle a_i = \frac{1}{i}$

4. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^2}$

   If the last series converges, then find an estimate (i.e. partial sum) to within $.01 = \frac{1}{100}$.

5. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1+n}{2n}\right)^n$ Hint: Limit compare with $\sum\left(\frac{1}{2}\right)^n$. To do so, factor out $\left(\frac{1}{2}\right)^n$ from $\left(\frac{1+n}{2n}\right)^n$ to cancel. Also use $\lim_{n\to\infty}\left(\frac{1+n}{n}\right)^n = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$.