9.5 Power Series Group Work Target Practice

$$\sum_{n=1}^{\infty} \frac{(5x)^n}{n} = 5x + \frac{25x^2}{2} + \frac{125x^3}{3} + \dots$$

1. What is the center, the value of x that makes this power series 0 and hence gives convergence?

   $x = 0$

   A power series always converges at the center, and we look to see how far away from it we can go to still converge. For a power series, either geometric series tests or the ratio test is helpful to find the radius and interval of convergence.

2. Is the series geometric? No. The ratio of the terms is not constant: $\frac{125x^3}{3} / \frac{25x^2}{2} = \frac{10x}{3} \neq \frac{25x^2}{2} / 5x = \frac{5x}{2}$

3. Use the ratio test, where we look at the ratio of successive terms to see if it is bounded (convergence for $L < 1$) or unbounded (divergence for $L > 1$). The test fails if $L = 1$.

   $$L = \lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} \frac{|\frac{(5x)^{n+1}}{n+1}|}{|\frac{(5x)^n}{n}|} = \lim_{n\to\infty} \frac{|\frac{(5x)^{n+1}}{(5x)^n}|}{|\frac{n+1}{n}|} = \lim_{n\to\infty} \frac{|5x|}{|\frac{(n+1)}{n}|} = \lim_{n\to\infty} |5x| \frac{n}{n+1}.$$

   To cancel, we are using the law of exponents, that $\dfrac{(5x)^{n+1}}{(5x)^n} = (5x)^{n+1-n} = (5x)^1 = 5x$.

   Since ratio of $\lim_{n\to\infty} |5x| \dfrac{n}{n+1}$ tends to $\frac{\infty}{\infty}$ as n gets large, we use L'Hôpital's Rule to take the derivative of the top and bottom with respect to n. So we see that $\lim_{n\to\infty} \dfrac{|a_{n+1}|}{|a_n|} = \lim_{n\to\infty} |5x| \dfrac{1}{1} = |5x|$.

   For the power series to converge $|5x| = L < 1$, so $|x| < \frac{1}{5}$. Hence R, the radius of convergence, is $\frac{1}{5}$.

4. What is the radius of convergence R? $\frac{1}{5}$

5. Write the open interval about the center where the power series converges.

   The center is 0 and $R = \frac{1}{5}$, so the open interval about the center where the power series converges is obtained by adding and subtracting $R = \frac{1}{5}$ from the center: $(0 - \frac{1}{5}, 0 + \frac{1}{5}) = (-\frac{1}{5}, \frac{1}{5})$

6. To find the interval of convergence, first check the left endpoint. Write the series given by the left endpoint and check whether that series converges or diverges:

   The left endpoint is $x = -\frac{1}{5}$ and we plug this into $\displaystyle\sum_{n=1}^{\infty} \frac{(5x)^n}{n}$: $\displaystyle\sum_{n=1}^{\infty} \frac{(5(-\frac{1}{5}))^n}{n} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n}$.

   This is an alternating series with decreasing terms and it converges by the alternating series test.

7. Next check the right endpoint. Write the series given by the right endpoint and check whether that series converges or diverges:

   The right endpoint is $x = \frac{1}{5}$ and we plug this into $\displaystyle\sum_{n=1}^{\infty} \frac{(5x)^n}{n}$: $\displaystyle\sum_{n=1}^{\infty} \frac{(5\frac{1}{5})^n}{n} = \sum_{n=1}^{\infty} \frac{1^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$.

   This is the harmonic series and it diverges by the integral test.

8. Write the interval of convergence: $[-\frac{1}{5}, \frac{1}{5})$