

Maple and Solutions of Differential Equations

Goal: Explore differential equations using a variety of representations (numerical, symbolic, and graphical) through pattern exploration assisted by appropriate technology, including the computer algebra system Maple, which is one of the course goals.

Read the text and hit return in each Maple command line (the commands are in red).

1. By-hand solution of $y'=y t$

Solve the separable differential equation $y'=y t$ with the initial condition $y(0) = .5$. Show work to solve for y , including where you use the initial condition.

$y'=y t$ is equivalent to $dy/dt = y t$. To separate the variables we put the y components on one side and the t components on the other. Then $dy/y = t dt$. Next we integrate $\ln|y| = \frac{t^2}{2} + C$

To solve for y we must use the inverse function of $\ln(x)$, which is e^x . So $|y| = e^{\frac{t^2}{2} + C} = e^{\frac{t^2}{2}} e^C = C I e^{\frac{t^2}{2}}$

The initial condition tells us that $y(0) = .5$ so $.5 = C I e^0 = C I$. Thus $|y| = 0.5 e^{\frac{t^2}{2}}$

2. Execute the following commands for the general solution, and then the solution with the initial condition.

```
> with(DEtools): with(Student[NumericalAnalysis]):  
dsolve(diff(y(t),t)=y(t)*t);  
dsolve({diff(y(t),t)=y(t)*t, y(0)=.5});
```

$$y(t) = _C I e^{\frac{t^2}{2}}$$

$$y(t) = \frac{e^{\frac{t^2}{2}}}{2}$$

(1)

Maple can quickly solve DEs numerically, graphically and algebraically!

3. Comparing Maple and by-hand work.

Compare your by-hand solution in 1. to Maple's solution in 2. and resolve any differences.

It is the same. Maple expressed 0.5 as 1/2.

4. Sketch slope field tick marks

At the two points

$[0, 0.5]$ and

$[0.5, 0.5]$

solve for and sketch slope field tick marks,

To find the direction of the tick marks, we plug the points into the right hand side of the DE, so that we see what the slope is at the point:

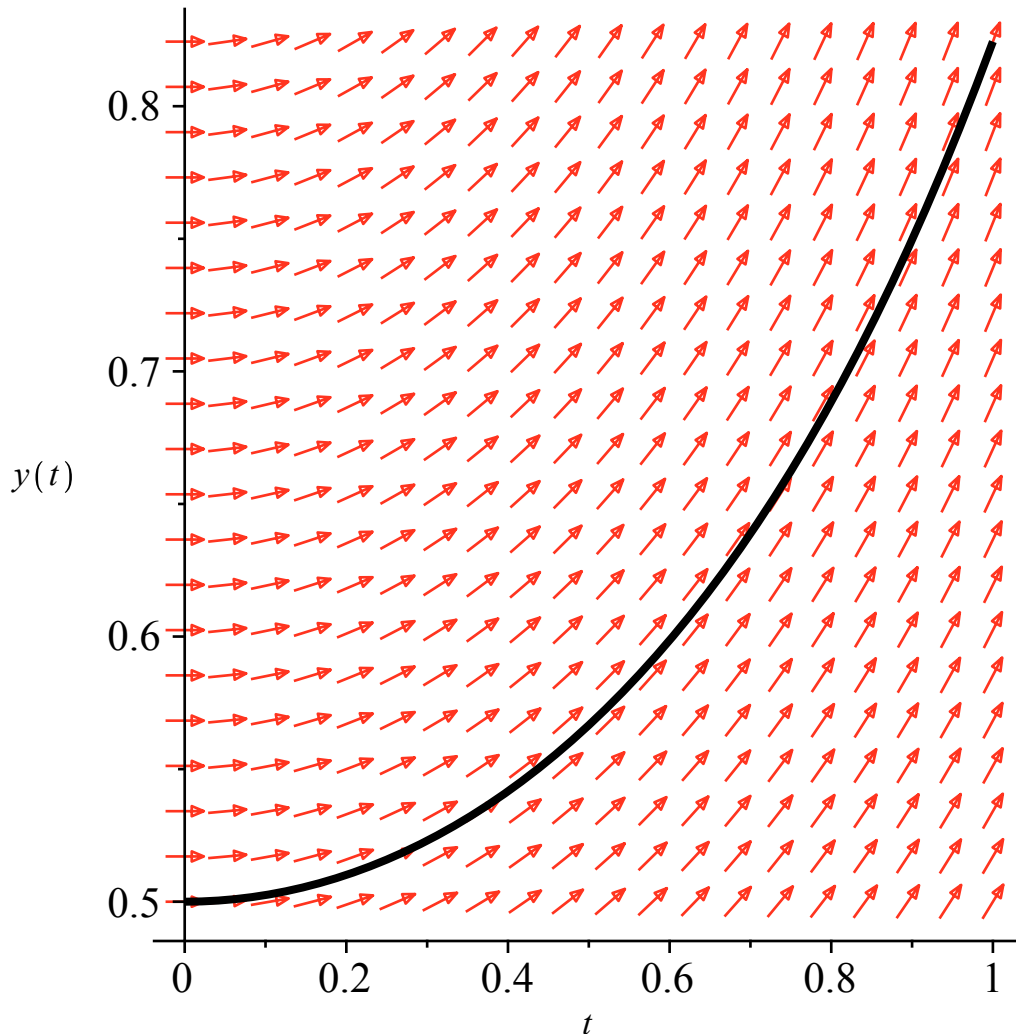
$[0, 0.5]$ $y'=y t$ becomes $y'=0.5 \cdot 0 = 0$, so the tick mark is horizontal.

$[0.5, 0.5]$ $y'=y t$ becomes $y'=0.5 \cdot 0.5 = .25$, so the tick mark is slanted up to the right a bit.

5. Compare your slope field markings with Maple's.

Execute the following command and then compare what you drew with Maple's drawing, which should agree at the two points.

```
> DEplot(diff(y(t),t) = y(t)*t, y, t = 0 .. 1, [y(0) =.5], arrows = medium, linecolor = black);
```



Yes there is agreement of a horizontal slope at $[0, 0.5]$ and slanted up to the right a bit at $[0.5, 0.5]$.

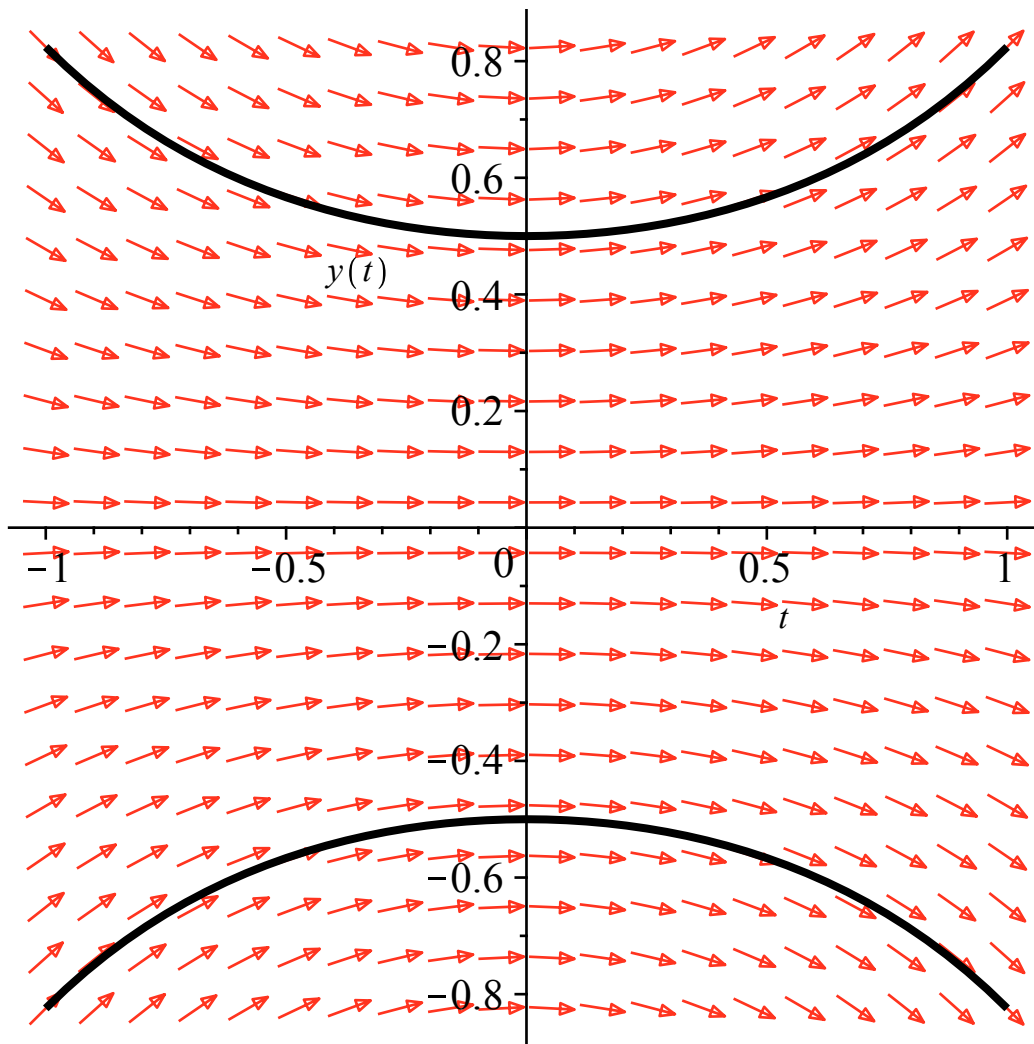
6. Equilibrium solutions

Does this DE have any equilibrium solutions?

If we set the DE $y'=y t$ equal to 0 to see what constant y values can work, we see $y'=y t=0$ shows $y=0$.

This equilibrium is neither unstable nor stable as you can see from the following plot:

```
> DEplot(diff(y(t),t) = y(t)*t, y, t = -1 .. 1, [y(0) =.5, y(0)=-.5], arrows = medium, linecolor = black);
```



7. Apply Euler's method two times

Use Euler's method, applied twice, starting at the point $[0, .5]$ with $\Delta t = .5$

t	y	y'	$\Delta y = \text{slope} \Delta t$	$(t + \Delta t, y + \Delta y)$
0	0.5	$y' = y \ t = 0.5 * 0 = 0$	0	$(0 + .5, 0.5 + 0)$
0.5	0.5	$0.5 * 0.5 = .25$	$.25 * .5 = .125$	$(.5 + .5, .5 + .125) = (1, .625)$

8. Underestimate or overestimate

Using the slope field graph of the solution through $[0, .5]$ in #5, will Euler's method be an underestimate or overestimate?

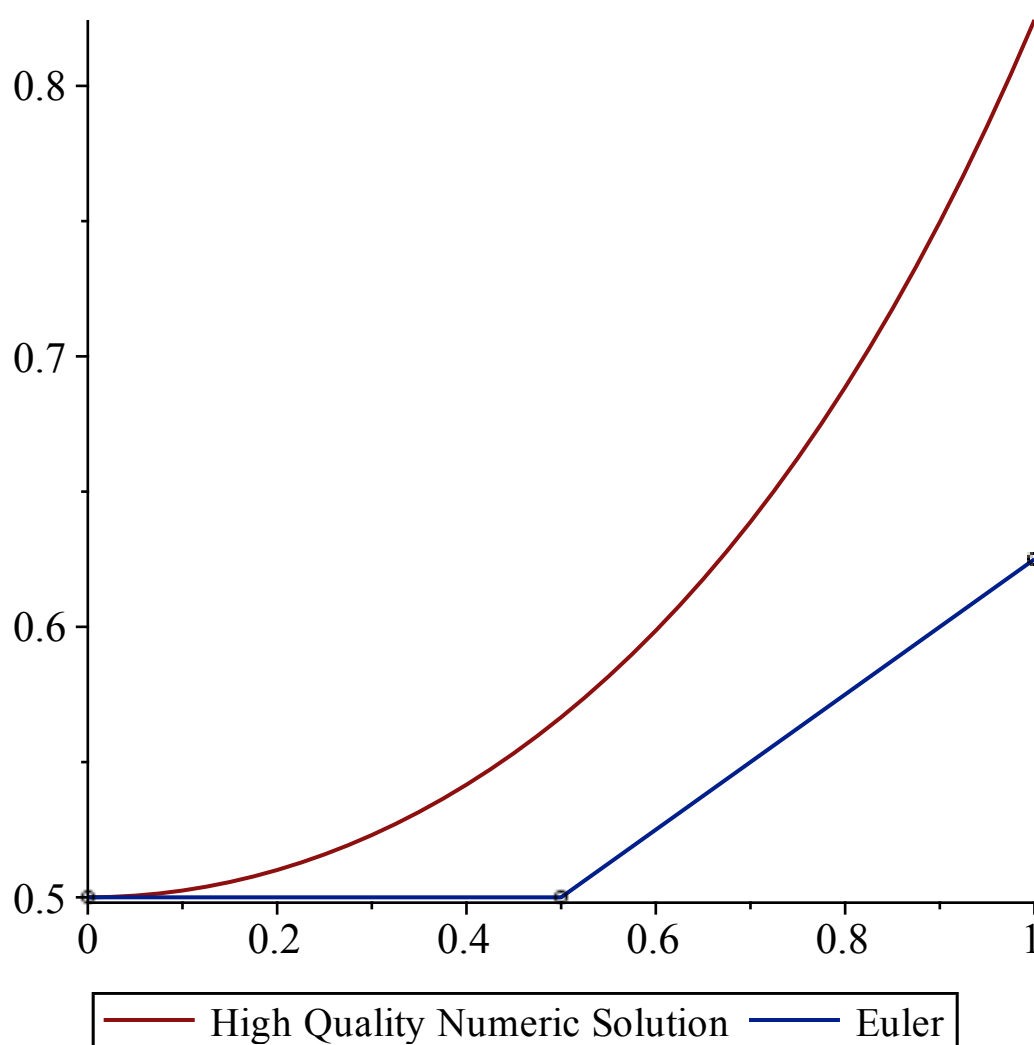
Euler's method will be an underestimate since the linear tick mark is below the solution.

9. Euler's method in Maple

Execute Maple's commands to generate graphical and numerical representations of Euler's method.

Graphical:

```
> Euler(diff(y(t),t) = y(t)*t,y(0)=0.5,t=1, output=plot, numsteps=2);
```



Numerical:

```
> Euler(diff(y(t),t) = y(t)*t,y(0)=0.5,t=1,numsteps=2);
0.6250
```

(2)

10. Comparing Maple and by-hand work.

Compare your by-hand solution in #7 to Maple's solution in #9 and resolve any differences.

They agree with the second step being [1, .625]

11. Growing or shrinking proportional to the amount present

Write a separable differential equation showcasing when the rate of change is proportional to the amount present. Select a specific rate of change, and then repeat some of the above for this DE. Notice that multiplication in Maple is * inside the commands and **you will need to modify the DE by changing =y(t)*t to your new DE in all the commands** below:

The general solution and the solution with the initial condition

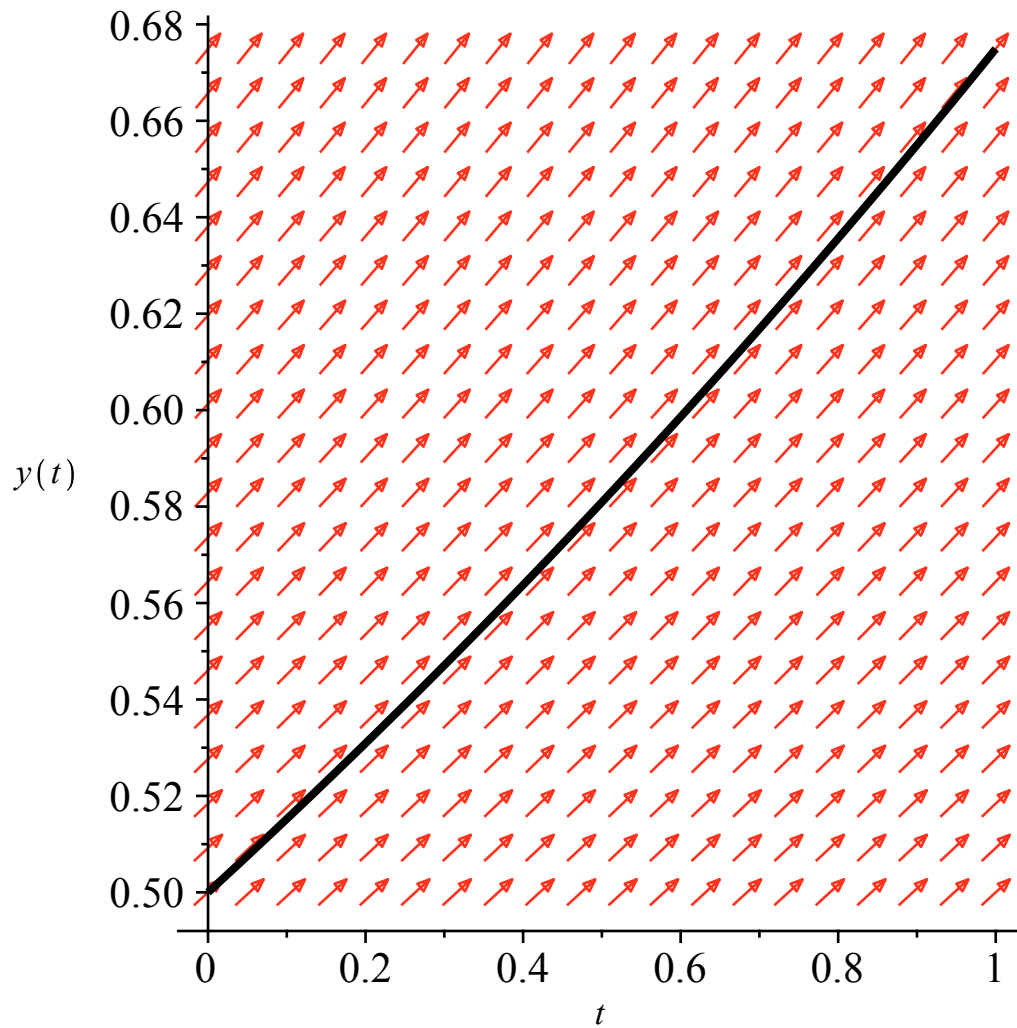
```
> dsolve({diff(y(t),t)=y(t)*.3, y(0)=.5});
```

$$y(t) = \frac{e^{\frac{3t}{10}}}{2}$$

(3)

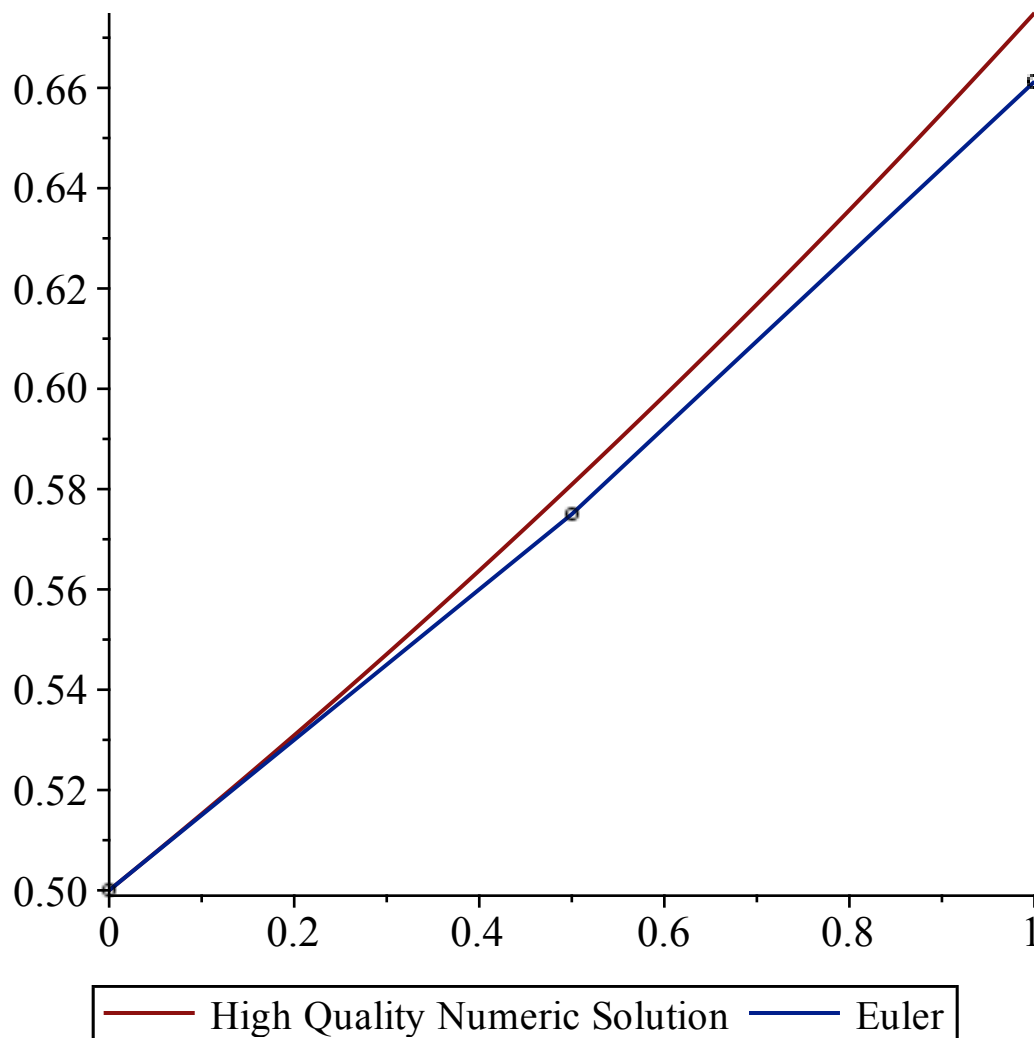
Slope field

```
> DEplot(diff(y(t),t) = y(t)*.3, y, t = 0 .. 1, [y(0) =.5], arrows =  
medium, linecolor = black);
```



Euler's method

```
> Euler(diff(y(t),t) = y(t)*.3,y(0)=0.5,t=1, output=plot, numsteps=2)  
;
```



```
> Euler(diff(y(t), t) = y(t)*.3, y(0)=0.5, t=1, numsteps=2);
0.6612
```

(4)

Question: At the point [0, .5], will Euler's method be an underestimate or overestimate?
An underestimate since the linear approximation is below the solution.

12. Newton's Law of Colling or Heating

Next write a separable differential equation representing Newton's Law of Cooling or Heating. Select some specific numbers for the constant of proportionality and the ambient temperature for this DE, and then repeat the above for this DE. Notice that multiplication in Maple is * inside the commands and **you will need to modify the DE by changing =y(t)*t to your new DE in all the commands** below:

The general solution and the solution with the initial condition

```
> dsolve({diff(y(t), t)=-.5*(y(t)-72), y(0)=.5});
```

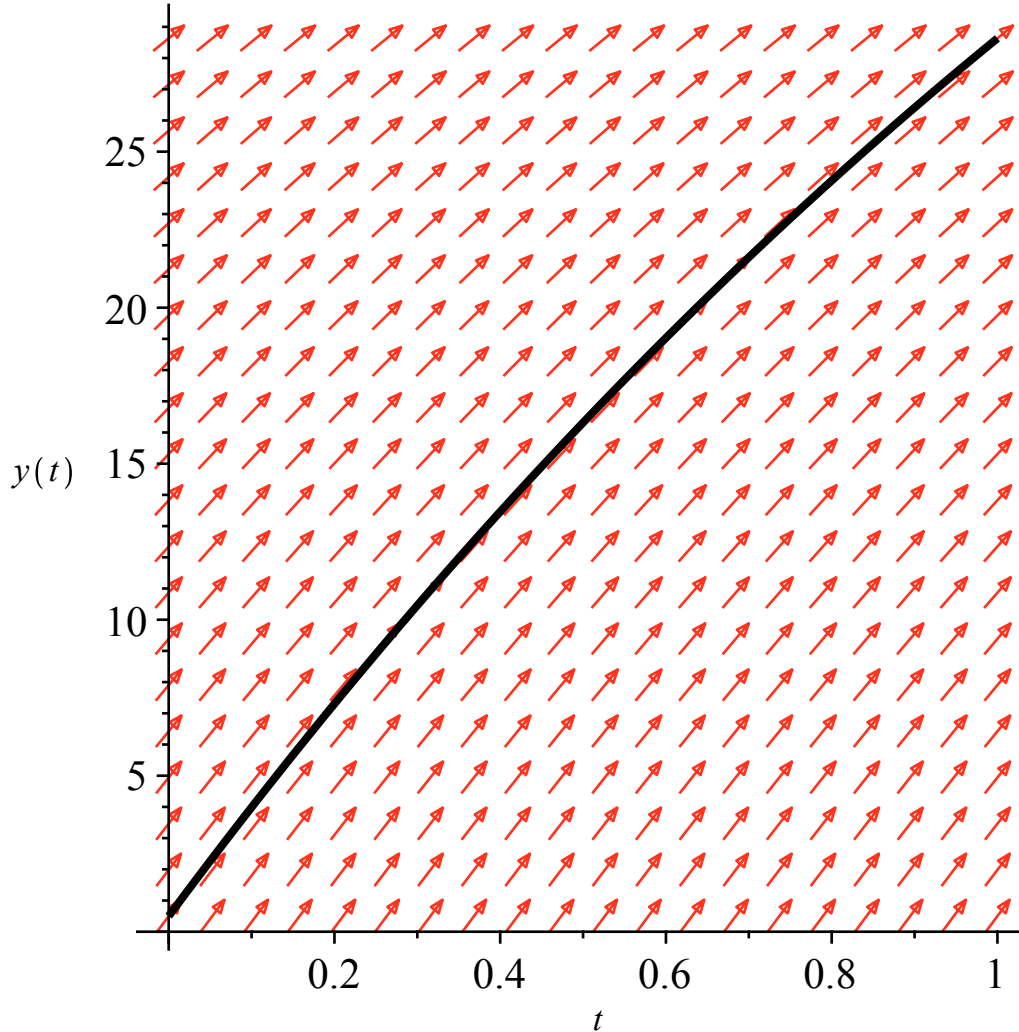
$$y(t) = 72 - \frac{143 e^{-\frac{t}{2}}}{2}$$

(5)

To do by hand: Solve the separable differential equation by-hand and compare with Maple's output

Slope field

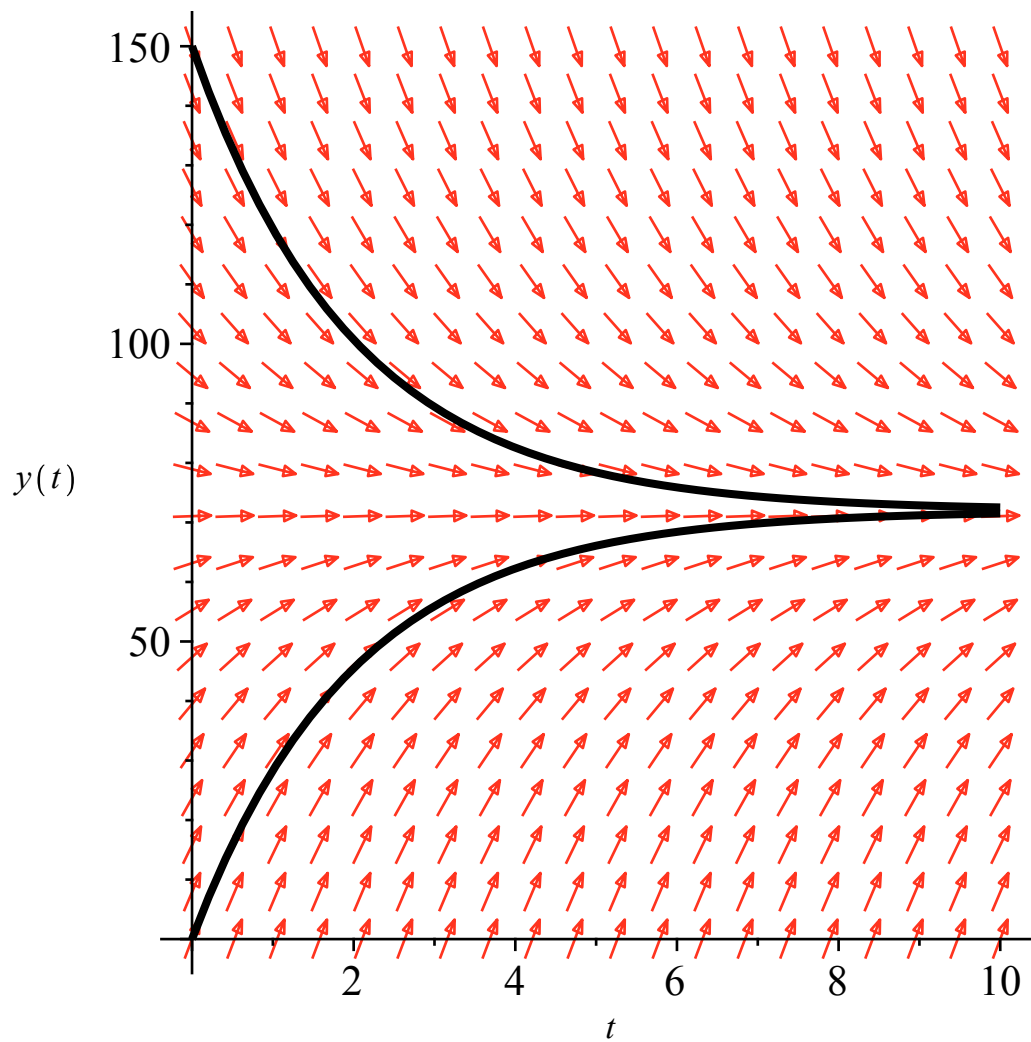
```
> DEplot(diff(y(t),t) = -.5*(y(t)-72), y, t = 0 .. 1, [y(0) =.5],  
arrows = medium, linecolor = black);
```



Question: Does this DE have any equilibrium solutions? If so are they stable, unstable, or neither?

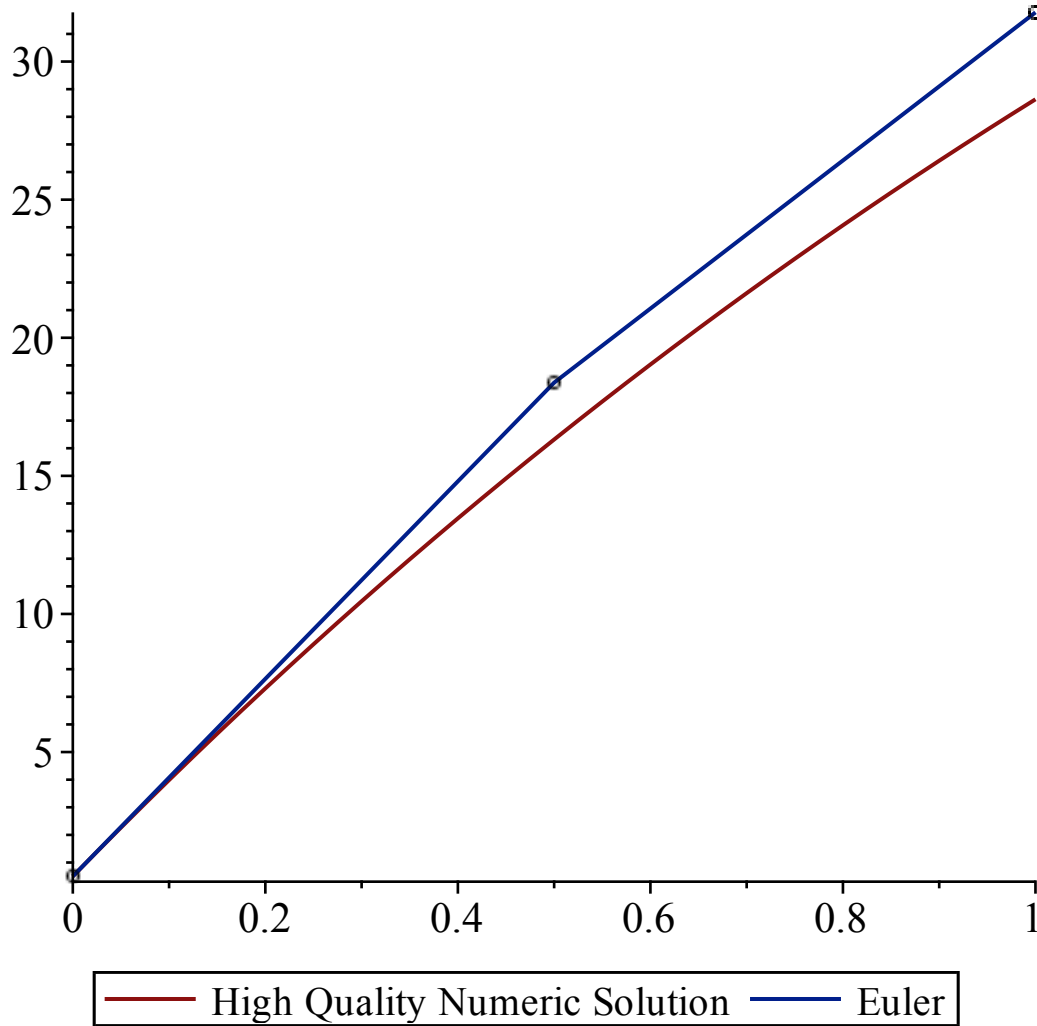
If we set $y' = -.5*(y(t)-72)$ equal to 0, we see that $y=72$ is a solution. Given the context of the problem, it is a stable solution as any other temperatures will head towards 72---the negative constant of proportionality means that objects above 72 will cool to reach that value, and objects below 72 will heat to reach that value. We can see this in the graph below:

```
> DEplot(diff(y(t),t) = -.5*(y(t)-72), y, t = 0 .. 10, [y(0) =0, y(0)  
=150], arrows = medium, linecolor = black);
```



Euler's method

```
> Euler(diff(y(t),t) = -.5*(y(t)-72),y(0)=0.5,t=1, output=plot,  
numsteps=2);
```

```
> Euler(diff(y(t), t) = -.5*(y(t)-72), y(0)=0.5, t=1, numsteps=2);
      31.78
```

(6)

Question: At the point [0, .5], will Euler's method be an underestimate or overestimate?
 An overestimate since the linear approximation is above the solution.