

Introduction to Maple Activities and Questions

Maple is a computer algebra system, which was first developed in 1980 out of the University of Waterloo in Canada.

Goal: Maple can handle numerical, symbolic, and graphical representations. Exploring these representations is one of the course goals of Calculus and Analytic Geometry II, as is an introduction to this kind of software to get acclimated with its benefits, and use it to help us make connections to the work we are doing by-hand. Calculus II offers many opportunities for creative problem solving. One goal of group work and interactions in class and lab is to help you practice and get used to productive failures (where you understand and correct mistakes) so that you can become independent and successful yourself.

So engage with the material and your neighbors as you write your responses on a piece of paper.

In the following, **execute the command code by hitting return on each red line**, as you read and answer questions.

Maple knows all the basic techniques of integration that we are supposed to learn, including substitution, integration by parts and much more.

Maple applies these techniques without telling us which one it is using and comes up with a final answer, when it can.

Int displays the integral while int computes it.

Question 1: Is the following a substitution, parts, or neither? (go to the red line and enter return to execute the command and use it to help you)

> **Int(x^3*ln(x), x); int(x^3*ln(x), x);**

$$\int x^3 \ln(x) dx$$
$$\frac{x^4 \ln(x)}{4} - \frac{x^4}{16} \quad (1)$$

Please note that it should be + c at the end, but Maple leaves that off!

This is integration by parts because it is a product of two different types of functions (algebraic and log) where w-subst doesn't already work.

Question 2: If the above integral is integration by substitution, write down w , dw , and the integral with respect to w . If it is integration by parts, write down u , u' , v , v' , and $uv - \int u'v dx$.

Detail says to make v' the first type of function that comes after the D. In this case "a" algebraic, comes before "l" ln.

So $u = \ln(x)$ and $v' = x^3$

Then $u' = \frac{1}{x}$ and $v = \frac{x^4}{4}$

$$uv - \int u'v dx = \frac{x^4 \ln(x)}{4} - \int \frac{1}{x} \frac{x^4}{4} dx = \frac{x^4 \ln(x)}{4} - \int \frac{x^4 - 1}{4} dx = \frac{x^4 \ln(x)}{4} - \int \frac{x^3}{4} dx + c$$

I asked you to stop there but notice that if we evaluate the integral then this matches Maple's response

$$\frac{x^4 \ln(x)}{4} - \frac{x^4}{16} + c$$

Visualization of the definite integral

Maple can also compute the definite integral:

```
> Int(x^3*ln(x), x=0..1); int(x^3*ln(x), x=0..1);
```

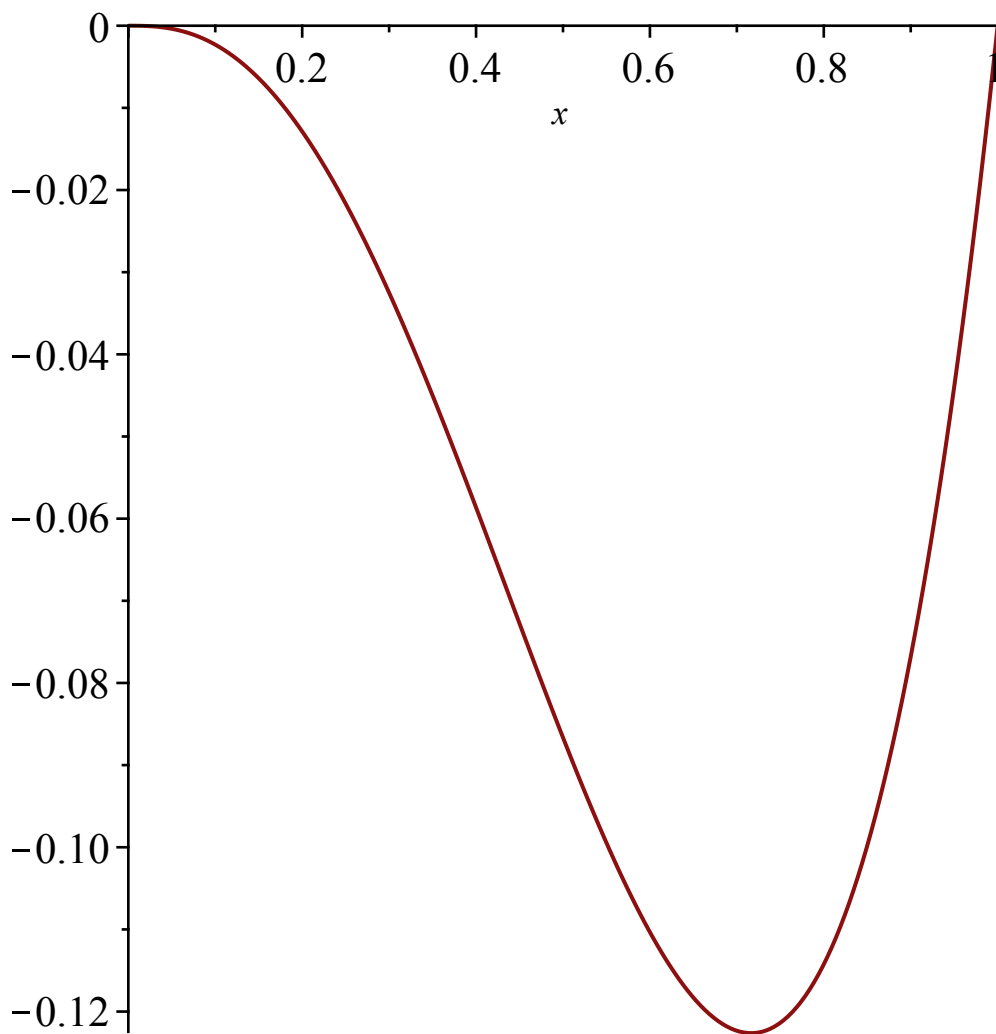
$$\int_0^1 x^3 \ln(x) dx$$

$$-\frac{1}{16}$$

(2)

We can also plot the function, and see the geometric representation of the area under the curve that Maple just solved for to see why Maple gives a negative response, as the curve is under the x-axis:

```
> plot(x^3*ln(x), x=0..1);
```



Question 3: Is the following a substitution, parts, or neither?

```
> Int(x^2*sin(x^3)*cos(x^3), x); int(x^2*sin(x^3)*cos(x^3), x);
```

$$\int x^2 \sin(x^3) \cos(x^3) dx$$

$$-\frac{\cos(x^3)^2}{6} \quad (3)$$

It is w -subs. This is a product of three functions rather than two that is typical for parts, so parts doesn't work, and we can see why w -subs works as below.

Question 4: If the above integral is integration by substitution, write down w , dw , and the integral with respect to w . If it is integration by parts, write down u , u' , v , v' , and uv - integral $u'v dx$.

$w = \cos(x^3)$ works here. While $w = x^3$ might be a natural first choice, this doesn't work, because we don't have only the differential in the integral (up to a constant). It leads to an integral we can't evaluate.

Using Maple's output we can see that Maple did something with $\cos(x^3)$ since the integral becomes

$$-\frac{\cos(x^3)^2}{6}. \text{ We can see that } w = \cos(x^3) \text{ works here. Then } dw = -\sin(x^3) 3x^2 dx,$$

which we have except the constant. So $\frac{dw}{-3} = \sin(x^3) x^2 dx$.

$$\int x^2 \sin(x^3) \cos(x^3) dx = \int \cos(x^3) \sin(x^3) x^2 dx = \int w \frac{dw}{-3}$$

I asked you to stop there, but notice that if we evaluate this integral then we have $-\frac{1}{3} \frac{w^2}{2} + c$, which

$$\text{is } -\frac{1}{3} \frac{\cos(x^3)^2}{2} + c = -\frac{\cos(x^3)^2}{6} + c$$

so this matches Maple's response.

Question 5: The following integral is quite important in statistics and other real-life applications such as for harmonic oscillators. It is known as the Euler–Poisson integral or the Gaussian integral. This is not an elementary integral. Write down what Maple outputs for the integral?

> **Int(exp(-x^2), x); int(exp(-x^2), x);**

$$\int e^{-x^2} dx$$

$$\frac{\sqrt{\pi} \operatorname{erf}(x)}{2} \quad (4)$$

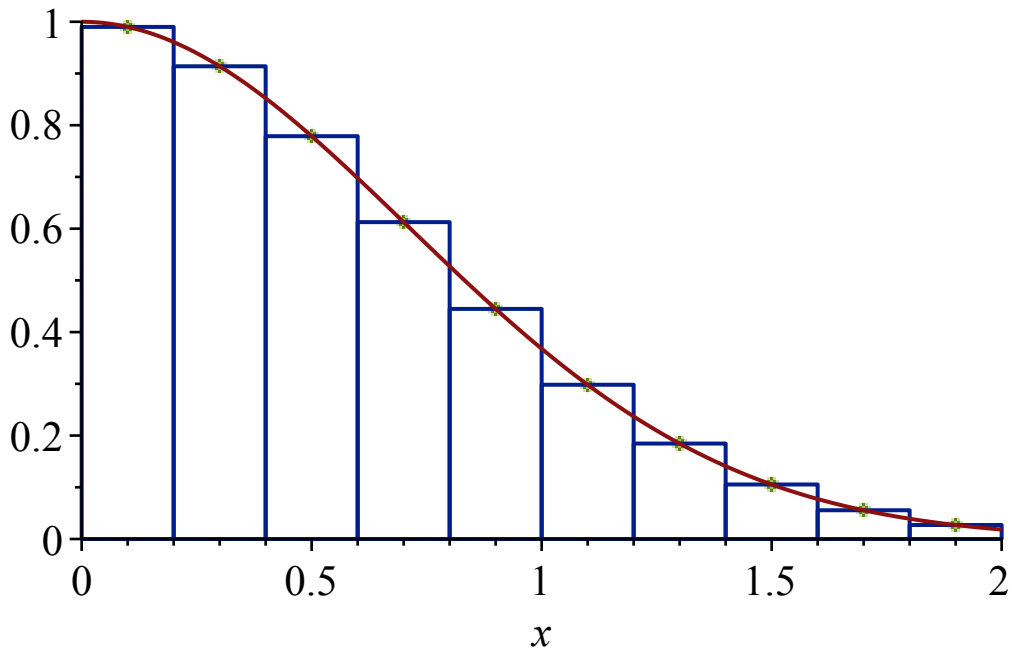
If you search for erf(x) on the internet you would notice that it is defined as a constant times the integral, so the integral has erf in it and erf has the integral in it, i.e. not all that revealing.

To find more information about it, Maple can approximate integrals it can not compute directly by Riemann sum approximations (like you did in Calculus I) or by Taylor series, both of which we will be exploring this semester..

Execute the following and notice the rectangles that have width Δx and height as the function value.

$\int f(x) dx$ arises as Δx goes to 0 and we get better approximations that approach the integral.

> **with(Student[Calculus1]): ApproximateInt(exp(-x^2), x = 0 .. 2, output = plot);**



A midpoint Riemann sum approximation of $\int_0^2 f(x) dx$,

where $f(x) = e^{-x^2}$ and the partition is uniform. The approximate value of the integral is 0.8822020699. Number of subintervals used: 10.



Next go back to the class highlights page and follow the instructions there.