Taylor Polynomials in Maple

Take notes on paper as you hit return at the end of each Maple command line (the commands are in red).

```
> with(Student[Calculus1]): with(plots):
```

Taylor polynomials for sin(x) about x=0

Maple can compute Taylor polynomials of degree n very fast. Here n=15, and we are computing the Taylor polynomial for sin(x) about x=0. Execute and then write the first few terms in your notes:

```
> TaylorApproximation(sin(x), x = 0, order=15);
```

Notice that instead of writing the Taylor series using factorial, like 3!, Maple multiplies out the factorials.

We can also plot in Maple. First plot the linear approximation (Taylor polynomial of degree 1), and then plot of the 15th degree Taylor polynomial approximation, and then sketch them in your notes:

```
> TaylorApproximation(sin(x), x = 0, order=1,output=plot);
```

```
> TaylorApproximation(sin(x), x = 0, order=15,output=plot);
```

Where did f(x) go here? Well it is such a good approximation at that point, that the difference between the polynomial and the function is not observable near x=0 within this small (local) plotting area. The functions would separate further away from 0.

Taylor polynomials for sin(x) about x=1

Want to change what x=a is? No problem. Here is the 2nd degree polynomial for sin(x) about x=1. Write the polynomial in your notes.

```
> TaylorApproximation(sin(x), x = 1, order=2);
```

Where does the 1/2 coefficient of sin(1) come from? Well Maple collects terms.

By hand activity: Compute the 2nd degree polynomial for sin(x) about x=1 by hand on paper:

$$f(1) + f'(1)\,(x-1) + f^{(2)}(1)\frac{1}{2!}\,(x-1)^2$$

Show work, be sure to foil it out, collect like terms, and then show that yours is (eventually) the same as Maple.

Let's compare with the plot. Execute and then sketch it in your notes.

```
> TaylorApproximation(sin(x), x = 1, order=2,output=plot);
```

Taylor polynomials for arctan(x) about x=0

Use Maple commands to find the 11th degree polynomial that approximates arctan(x) at x=0. Copy and paste a command from above and then modify the function. Then write a few terms in your notes.

By hand activity: Next, using your Maple responses from above, write the general summation form for the Taylor polynomial of degree n, like James Gregory did for this series back in 1671. Start the index at 0.

Now use Maple commands to compare with the plot, and then sketch the plot in your notes.

Taylor polynomial for exp(x) about x=5

```
> TaylorApproximation(exp(x), x = 5, order = 2);
```

By hand activity: In class we obtained $e^5 + e^5 (x-5) + \frac{e^5}{2!}(x-5)^2$. Expand the terms and collect like terms to show that this matches Maple's output.