Slicing for Volume, Density and Work Practice Sheet

Come on in to office hours when you can! Sometimes it makes all the difference if I can go over material with you one on one. For example, we can talk through solutions to any or all of the Wiley problems in chapter 8, you can work on problems online and ask me questions or talk through your process as you do so, or whatever you would find useful. I care about your understanding and am always happy to help.

1. **Cone (radius 2 ft and height 5 ft) standing on its tip**

   (a) What is the volume of one slice of the cone via slicing horizontally. Show reasoning and a picture.

   Solution:

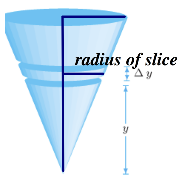
   

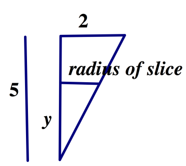
   cylindrical disk volume $= \pi$ radius of slice$^2 \Delta y$

   (b) Find the volume of the entire cone via slicing horizontally. Show reasoning and pics.

   Solution:

   Need to solve for the radius of the slice because we can't have two different variables (r and the y in Δy) in the Riemann sum or the integral.

   Similar triangles: $\frac{\text{radius of slice}}{y} = \frac{2}{5}$ so radius $= \frac{2y}{5}$

   Sub in: volume of a cylindrical disk slice 

   $= \pi$ radius of slice$^2 \Delta y = \pi (\frac{2y}{5})^2 \Delta y$

   Volume of the entire cone:

   y begins at the bottom of the cone at 0, and ends at the top of the cone at 5, so $\int_0^5 \pi (\frac{2y}{5})^2 dy$

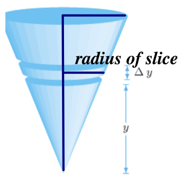
   (c) If the density $\delta(y)$ of the cone varies with it's height y, what is the total mass. Show reasoning.

   Slice perpendicular to y where $\delta(y)$ is approximately constant. Then slicing is as above in this example, so the same process would be followed to solve for the volume.

   mass $= \int_0^5 \delta(y)$ volume $= \int_0^5 \delta(y)\pi(\frac{2y}{5})^2 dy$

   (d) If the cone is partially filled with fresh water that weighs 62.5 lbs/ft^3 to a height of 4 ft, what is the total work required to pump the water out over the top? Show reasoning and pics.

   F d = ($62.5 lb/ft^3 \times$ volume) $\times$ distance the slice must be displaced



Since the height of the cone is 5, the displacement needed for a slice at height y is $5 - y$

F d = ($62.5 lb/ft^3 \times$ volume) $\times$ distance the slice must be displaced

$\quad$ = ($62.5 lb/ft^3 \times$ volume) $\times (5 - y)$

so we must solve for the volume, which we did in part b.

$\quad$ = ($62.5 lb/ft^3 \times \pi(\frac{2y}{5})^2 \Delta y) \times (5 - y)$

The only remaining item is to figure out the integration limits. The water begins at the bottom, $y = 0$ and stops at $y = 4$, so those are our limits.

$$\text{Work} = \int_0^4 (62.5 \times (\pi(\frac{2y}{5})^2 \times dy) \times (5 - y) = \int_0^4 62.5\pi \frac{4y^2}{25}(5 - y)dy$$

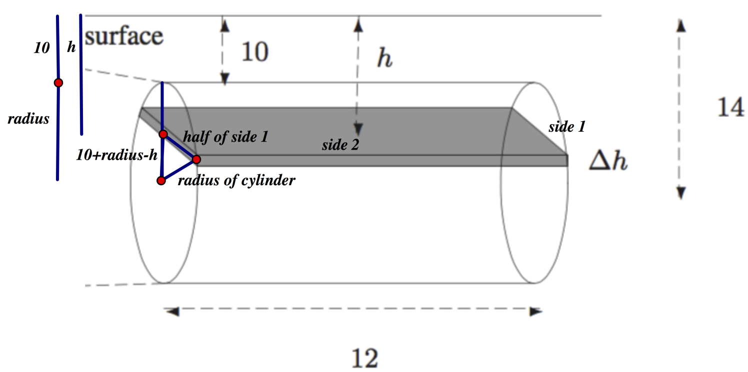
2. **Cylinder (radius 4 ft and length 12 ft) laying sideways**

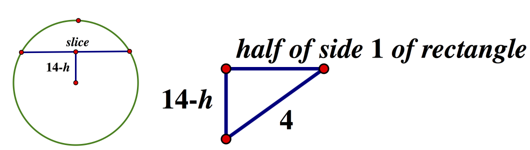
   (a) If the cylinder is buried 10 feet under ground, with the height h measured from ground level to a slice, and it is completely filled with salinated water, which weighs 64 lbs/ft^3 (more than fresh water which is 62.5 lbs/ft^3), what is the work required to pump the salinated water above ground? Show reasoning and pics.

   F d = ($64 lb/ft^3 \times$ volume) $\times$ distance the slice must be displaced

   $\quad$ = ($64 lb/ft^3 \times$ volume) $\times h$

   so we must solve for the volume:





   The volume of the rectangular box slice is length $\times$ width (side 1) $\times$ height

$$= 12 \times 2\sqrt{4^2 - (14 - h)^2} \times \Delta h$$

   F d = ($64 lb/ft^3 \times$ volume) $\times$ distance the slice must be displaced

   $\quad$ = ($64 lb/ft^3 \times$ volume) $\times h$

   $\quad$ = ($64 lb/ft^3 \times \left(12 \times 2\sqrt{4^2 - (14 - h)^2} \times \Delta h)\right) \times h$

Add up over the entire water supply. The water begins when h starts at 10 at the top of the tank, and goes to $10 + 2\times$ radius of the cylinder $= 10 + 4 + 4$ to the bottom of the tank.

Work $= \displaystyle\int_{10}^{18} (64 \times (12 \times 2\sqrt{4^2 - (14-h)^2} \times dh) \times h$

You try it!

3. **Cylinder (radius 3 ft and length 8 ft) laying sideways.** Answer the first three questions you did for the cone for the cylinder.

4. **Sphere (radius 5 ft)**. Answer the same questions you did for the cone for the sphere.

5. **Pyramid (radius 5 ft and height 20 ft)**. Answer the same questions you did for the cone for the pyramid.