8.4 Density and 8.5 Work

1. 8.4 Density, Mass and Volume Connections

   (a) Slice so density is approximately constant and sketch a picture of the resulting slice.

   (b) Which is the infinitesimal part of the slice? Circle: Δx or Δy or Δh or Δr

   (c) What is the length, area or volume we'll use (write it in that variable–the infinitesimal part is used here too)?
   if a length, like along a rod, it is the infinitesimal part, like Δx

   if an area like rectangle, where the width is the infinitesimal part, like Δx, and the height is a function $f(x)$, $g(x) - f(x)$, a constant, or the equation of a line $mx + b$
   new: area of annuli/rings $= \pi(r^2_{\text{outer}} - r^2_{\text{inner}}) = \pi(r+\Delta r)^2 - \pi r^2 = \pi r^2 + 2\pi r \Delta r + \pi \Delta r^2 - \pi r^2 = 2\pi r \Delta r + \pi \Delta r^2$, and when Δr is small Δr^2 goes to 0 faster, so we ignore that term in the limit, and use $2\pi r \Delta r$ for the area as we turn this into an integral

   if a volume many come from earlier in 8.1 or 8.2 (see those practice sheets), like box (length·width·height) or cylinder/disk ($\pi \cdot$ radius^2·height), where one of those lengths is the infinitesimal portion

   (d) The Riemann sum turns into an integral $\int \delta\cdot$ length or $\int \delta\cdot$ area or $\int \delta\cdot$ volume in that variable

2. 8.5 Work: Varying Force

   Work is force $\times$ distance displaced only applies if the force is constant while it is exerted over the distance. Integrals apply when we vary the force.

   (a) Slice so that the force is approximately constant on a slice for its displacement and sketch a picture of the resulting slice.

   (b) Which is the infinitesimal part of the slice? Circle: Δx or Δy or Δh or Δr

   (c) What is the displacement for that slice in terms of the slicing variable?

   (d) The Riemann sum is $\sum F\cdot$ displacement, like for Hook's Law to stretch (and hold) a spring, where $F(x) = kx$ is constant for displacement Δx and so the work is approximately $\sum F(x)\Delta x$
   We may need area or volume here too (see above), like
   $F =$ weight (force in lbs) $= 62.4$ lbs/ft$^3\times$ volume calculated as in 8.1 with
   work on a slice $= 62.4$ lbs/ft$^3\times$ volume $\times$ slice displacement

   (e) The Riemann sum turns into an integral $\int F\cdot$ displacement