8.4 Density, Mass and Volume Connections—builds on 8.1, 8.2

1. Slice so density is approximately constant and sketch a picture of the resulting slice.
2. Which is the infinitesimal part of the slice? Circle: Δx or Δy or Δh or Δr
3. What is the length, area or volume we'll use (write it in that variable–**the infinitesimal part too**)?

   if a length, like along a rod, it is the infinitesimal part, like Δx

   if an area like rectangle, where the width is the infinitesimal part, like Δx, and the height is a function $f(x)$, $g(x) - f(x)$, a constant, or the equation of a line $mx + b$

   or new: area of annuli/rings $= \pi(r^2_{\text{outer}} - r^2_{\text{inner}}) = \pi(r + \Delta r)^2 - \pi r^2 = \pi r^2 + 2\pi r\Delta r + \pi\Delta r^2 - \pi r^2 = 2\pi r\Delta r + \pi\Delta r^2$, and when Δr is small Δr^2 goes to 0 faster, so we ignore that term in the limit, and use $2\pi r\Delta r$ for the area as we turn this into an integral

   if a volume many come from earlier in 8.1 or 8.2 (see those practice sheets), like box (length·width·height) or cylinder/disk ($\pi \cdot$ radius^2·height), where one of those lengths is the infinitesimal portion

4. The Riemann sum turns into an integral $\int \delta\cdot$ length or $\int \delta\cdot$ area or $\int \delta\cdot$ volume in that variable

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