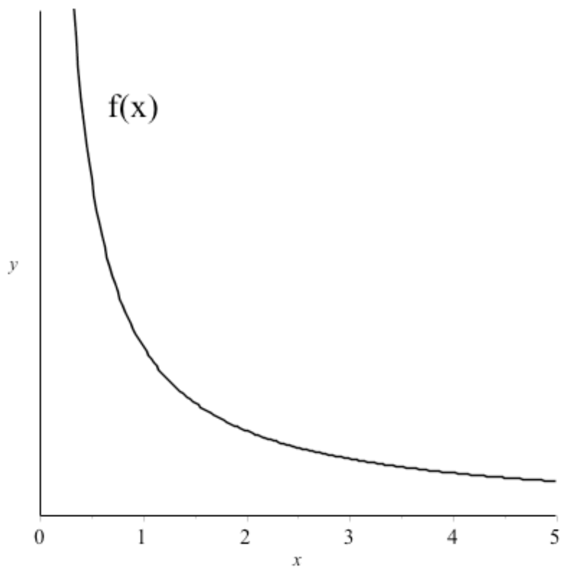
9.3 Integral Test

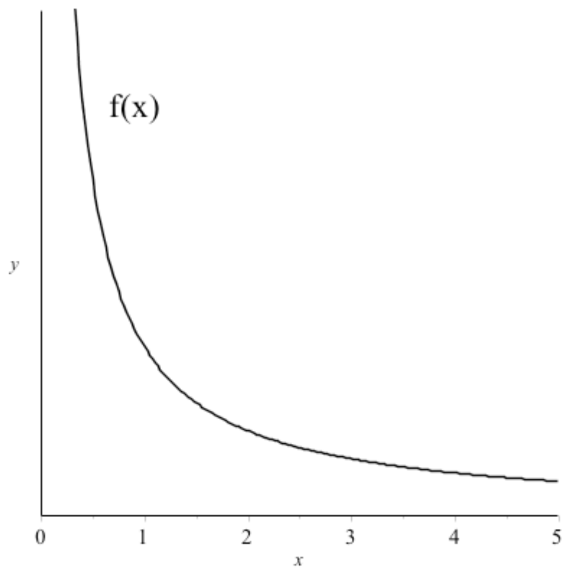
Goal: How do we determine if a series that isn't a geometric series converges?

Suppose $a_n = f(n)$, where $f(x)$ is decreasing and positive. We will consider $\sum_{n=1}^{\infty} a_n$.

1. **Draw the Left Sum** with $\Delta x = 1$ starting at $x = 1$



Draw the Right Sum with $\Delta x = 1$ starting at $x = 0$



2. In each box, put the element of the sequence a_n that gives the area of that box. Use the fact that $a_n = f(n)$ and $\Delta x = 1$.

3. What relationship between the improper integral $\int_1^{\infty} f(x)\, dx$ and the sum $\sum_{n=1}^{\infty} a_n$ is illustrated by the picture on the left?

   by the picture on the right?

4. Evaluate $\int_1^{\infty} \frac{1}{x}\, dx$.

5. Determine if $\sum_{n=1}^{\infty} \frac{1}{n}$ converges or diverges.

Example: Show $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

Examples: Do the following converge or diverge? If they converge, give bounds on the sum.

$$\sum_{n=1}^{\infty} \frac{1}{n^5}$$

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$$

$$\sum_{n=1}^{\infty} \frac{1}{e^{2n}}$$

$$\sum_{n=0}^{\infty} \frac{1}{1+n^2}$$

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^4}$$