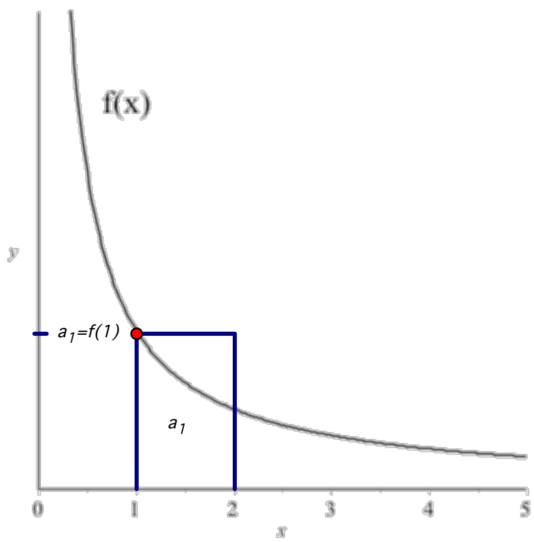
9.3 Integral Test

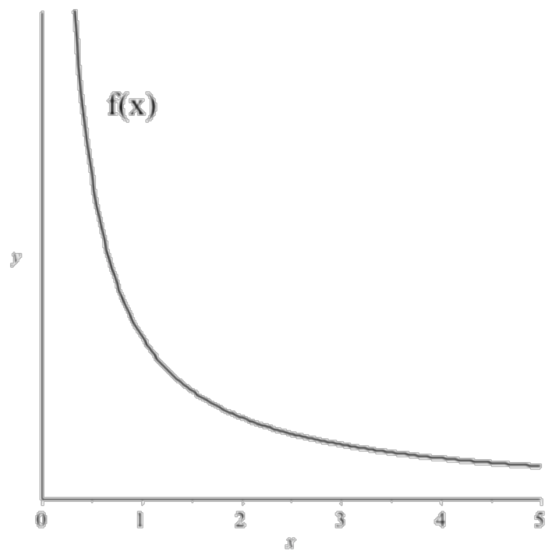
Goal: How can we use our prior improper integral work to help us understand certain series $\sum_{n=1}^{\infty} a_n$.?

1. Suppose $a_n = f(n)$, where $f(x)$ is decreasing and positive.
   In each box, put the element of the sequence a_n that gives the area of that box. Use $a_n = f(n)$ and $\Delta x = 1$.

   Draw the Left Sum with $\Delta x = 1$ starting at $x = 1$, and label each a_n

   Draw the Right Sum with $\Delta x = 1$ starting at $x = 0$, and label each a_n

2. What inequality relationship between the improper integral $\int_1^{\infty} f(x)\ dx$ and the sum $\sum_{n=1}^{\infty} a_n$ is illustrated by the picture on the left?

   by the picture on the right? (Hint: you'll need to bring in a_1 to the inequality)

3. Evaluate $\int_1^{\infty} \frac{1}{x}\ dx$.

4. Look at the integral test on the Series Theorems sheet. Determine if $\sum_{n=1}^{\infty} \frac{1}{n}$ converges or diverges.

Example: Use the integral test to show that $\sum_{n=1}^{\infty} \frac{1}{n^5}$ converges.

In fact, by extending our work on $\sum_{n=1}^{\infty} \frac{1}{n}$ and $\sum_{n=1}^{\infty} \frac{1}{n^5}$ we use the integral test on any fixed power of n:

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

9.1, 9.2 and 9.3 Group Work

For sequences EXPLAIN or SHOW WORK documenting why your answer is correct:

(a) does it converge or diverge, and why
(b) what is the limit if it converges?
(c) show work for L'Hôpital's Rule if it applies.

For series EXPLAIN or SHOW WORK documenting why your answer is correct:

(a) (LG 3) choose a series test we can successfully use on it from among geometric series, terms not going to 0, linearity, or integral test
(b) fully document why the series test works, including any assumptions
(c) specify whether the series converges or diverges, and why

$$\sum_{n=1}^{\infty} e^n$$

$$s_n = \frac{1}{e^{2n}}$$

$$\sum_{n=1}^{\infty} \frac{1}{e^{2n}}$$

$$\sum_{n=0}^{\infty} \frac{1}{1+n^2}$$

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^4}$$