## Partial Fractions in Maple

by Dr. Sarah
Maple is a computer algebra system, which was first developed in 1980 out of the at the University of Waterloo in Canada.

Maple can handle numerical, symbolic, and graphical representations. Exploring these representations and an exposure to this type of technology are course goals of Calc II.

## Example 1: Number 2 on target practice

In the following example, read through and execute the command code by hitting return at the end of the line. Compare with your by-hand work.
[> num2: $=(3 * x+11) /\left(x^{\wedge} 2-x-6\right)$;

$$
\begin{equation*}
\text { num } 2:=\frac{3 x+11}{x^{2}-x-6} \tag{1}
\end{equation*}
$$

Maple's convert feature factors the demonimator into parts and solves for the constants in the partial fractions:
[> convert(num2, parfrac, x);

$$
\begin{equation*}
\frac{4}{x-3}-\frac{1}{x+2} \tag{2}
\end{equation*}
$$

Maple's int feature integrates the function without specifying that it does it by partial fractions, although the natural logs in the output does give you a clue that it was via partial fractions.
[> Int(num2,x); int(num2,x);

$$
\begin{gather*}
\int \frac{3 x+11}{x^{2}-x-6} d x \\
4 \ln (x-3)-\ln (x+2) \tag{3}
\end{gather*}
$$

[Please note that it should be +c at the end, but Maple leaves that off!
Question 1: Aside from the +c , does Maple's work match your by-hand work?

## Example 2: Number 1 on target practice

EExecute the command code by hitting return at the end of each line.
>> num1:= 2/( $\left.s^{\wedge} 4-1\right)$;

$$
\begin{equation*}
\text { num } 1:=\frac{2}{s^{4}-1} \tag{4}
\end{equation*}
$$

= convert(num1, parfrac, s);

$$
\begin{equation*}
-\frac{1}{2(s+1)}-\frac{1}{s^{2}+1}+\frac{1}{2(s-1)} \tag{5}
\end{equation*}
$$

[Question 2: Your by-hand work should have looked like $\frac{A x+B}{s^{2}+1}+\frac{C}{s+1}+\frac{\mathrm{D}}{s-1}$
Use the above Maple output to specify what are the values of each of the four constants $\mathrm{A}, \mathrm{B}, \mathrm{C}$ and D in these generic forms of these numerators. (Hint: one constant will be 0 , while others will be fractions and/or negative numbers)

Question 3: Now that you have Maple's solution for the constants, finish off this problem--integrate each term in the partial fraction decomposition by-hand.
(Hint: arctan, ln and $w$-subsitution will be useful. If you are feeling comfortable with $w$-substitution then in this specific context you do not need to write out the substitution details)

Next execute the following integral command. Notice that it will differ from your by-hand integration!
[> Int(num1,s); int(num1,s);

$$
\begin{gather*}
\int \frac{2}{s^{4}-1} \mathrm{~d} s \\
-\operatorname{arctanh}(s)-\arctan (s) \tag{6}
\end{gather*}
$$

Question 4: Search the web for "Inverse Hyperbolic Tangent"and find a trig identity that relates it to logarithms (showing that your answer to Question 3 is equivalent to Maple's version)

Make sure I've gotten around to see your work when you've finished, and if there is any time left, you may work on Wileyplus homework for tomorrow, ask me questions, or leave. I'm happy to help!

