# Plotting in Maple for 7.6 Comparisons and Limits (Review of functions and their limits/asymptotes from prior classes)

**Goal**: Explore functions and their limits using graphical representations through pattern exploration assisted by appropriate technology in order to help with necessary prerequisite information for 7.6 and chapter 9.

Maple can create graphical representations very quickly!

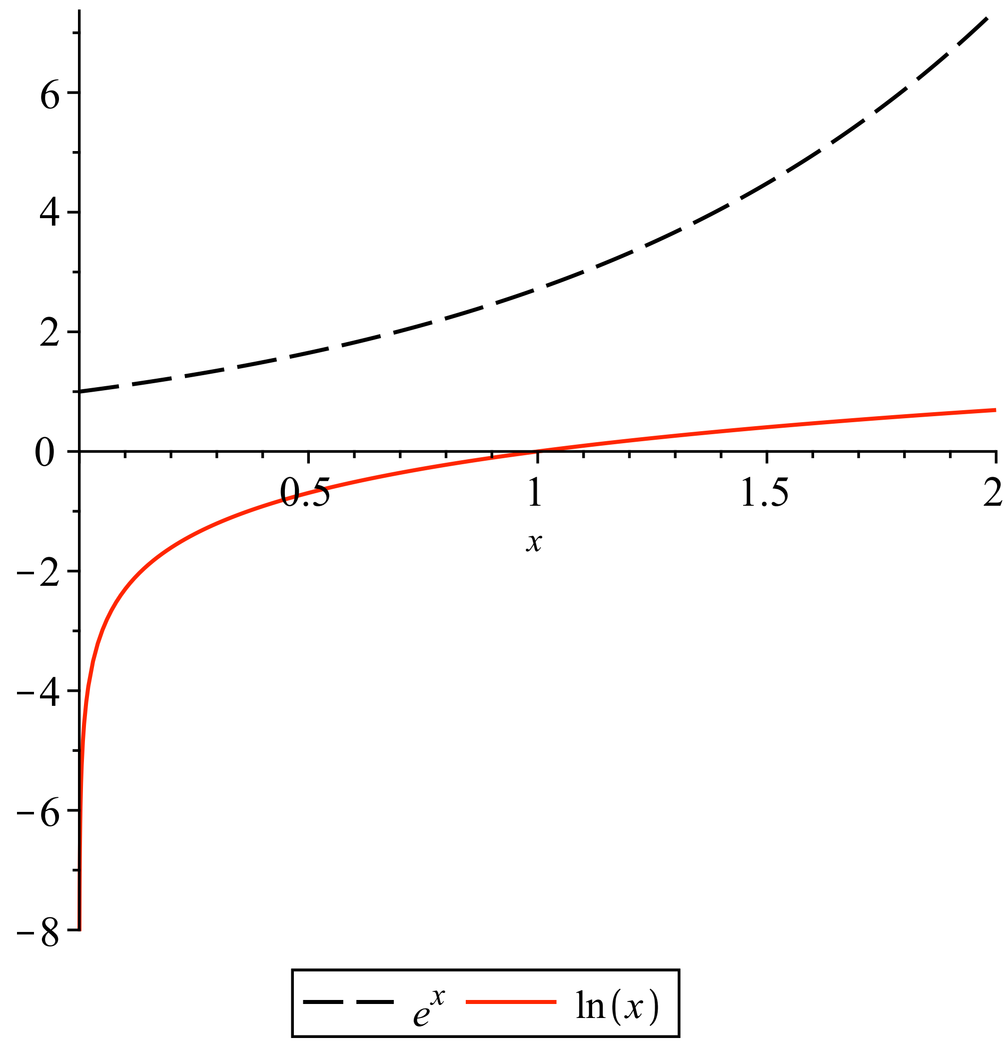
In the following execute the command code by hitting return in each red line.

The with(plots) command will open the plots package.

```
> with(plots):
```

## Example 1: exponential and natural log

```
> plot([exp(x),ln(x)],x=0..2, color=[black,red],legend=[typeset(e^x),
  typeset(ln(x))],linestyle=[dash,solid]);
```



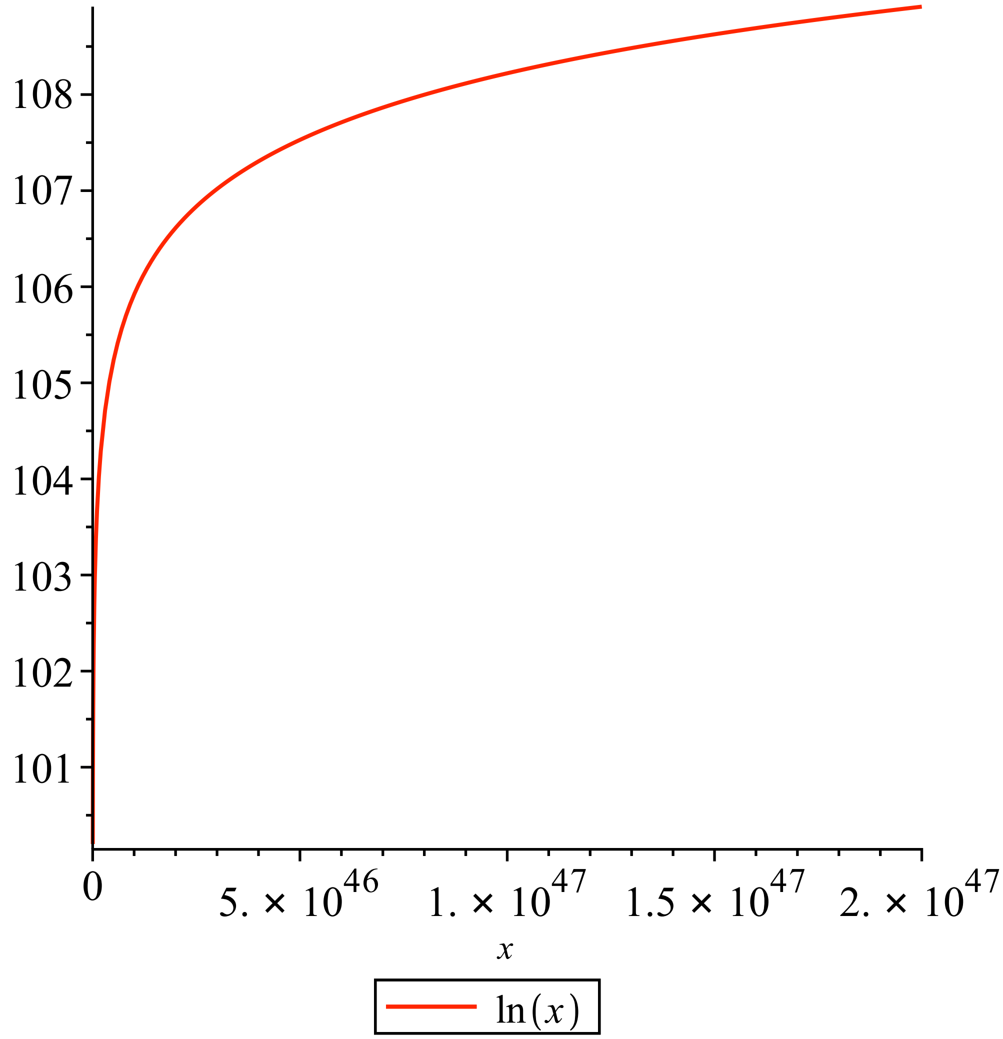

1) Roughly sketch them in your notes (to help you internalize the graphs of these functions) and

2) Specify the limits as *x* goes to infinity.
The limit for both functions is infinity.

If you aren't sure of  a limit from the given graph, you can remove focus on just one of the graphs.  For example, below I've removed exp(x) and removed typeset(e^x) from the command, leaving just ln(x).

**ACTIVITY**: Modify the 2 below to really large numbers to showcase that ln(x) continues to grow to infinity as x gets larger, just very slowly!

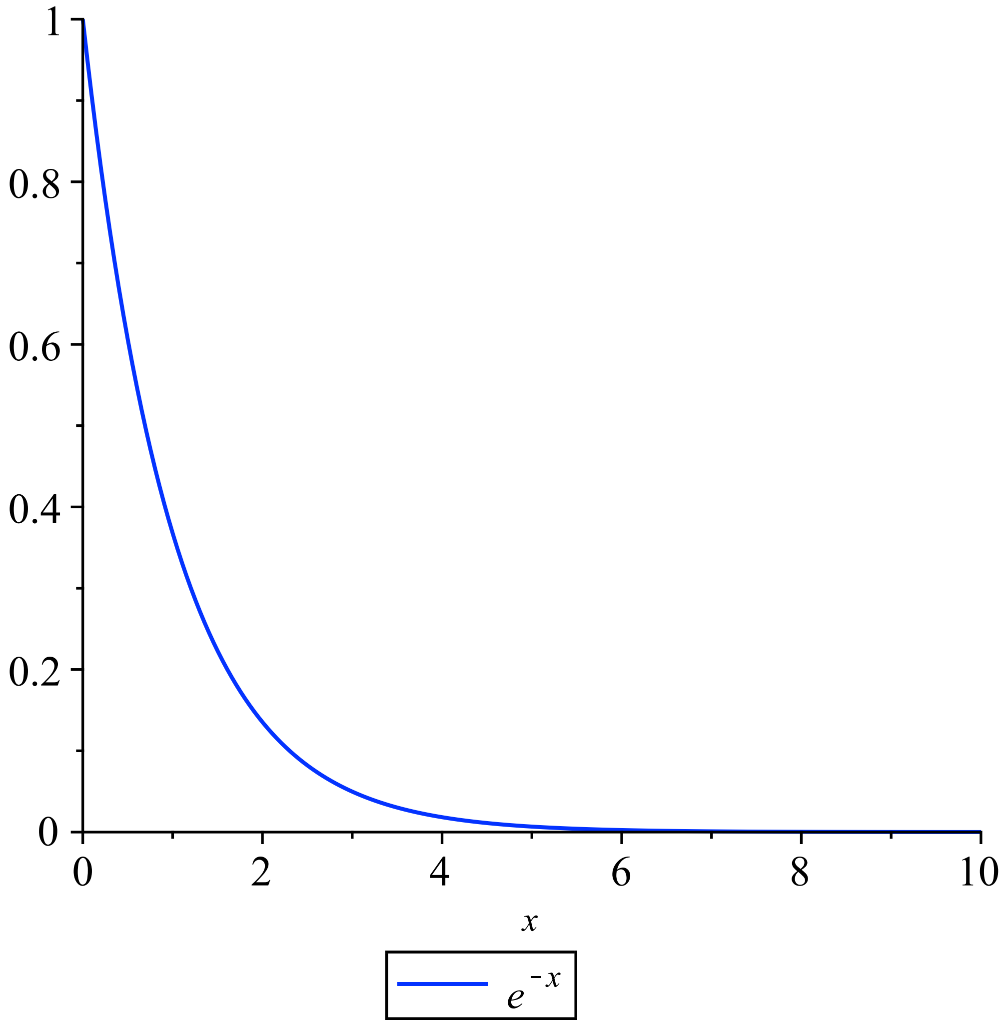
```
> plot([ln(x)],x=0..200000000000000000000000000000000000000000000000,
  color=[red],legend=[typeset(ln(x))],linestyle=[solid]);
```




## Example 2: functions to negative powers

1) Execute the command below and roughly sketch the graph in your notes and
2) Specify the limit as *x* goes to infinity.

```
> plot([exp(-x)],x=0..10,color=[blue],legend=[typeset(e^(-x))],
  linestyle=[solid]);
```

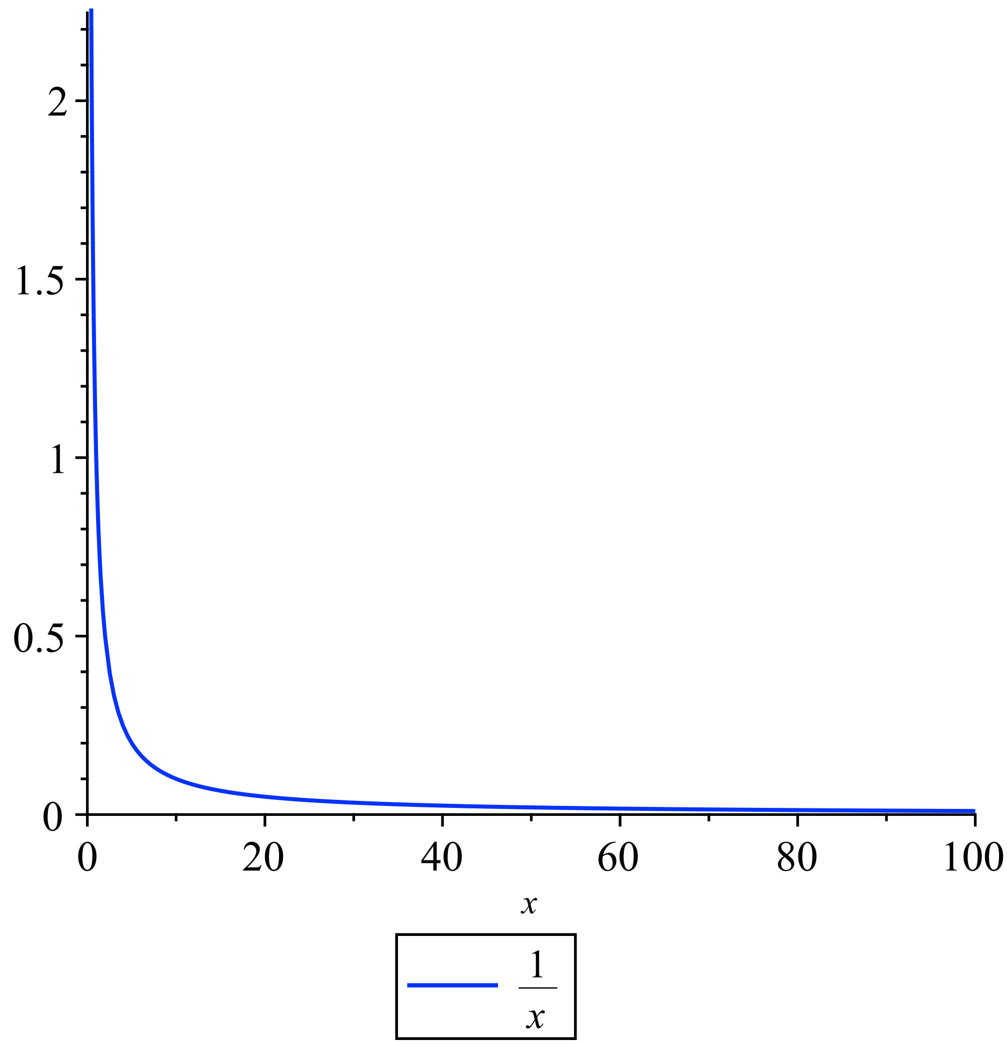



Since $e^{-x} = \frac{1}{e^x}$ we can also reason the limit algebraically, that as $x$ gets large, exponentials get large, so their reciprocals get small.
So the limit is 0.

3) What is the limit of $x^{-1}$ as $x$ goes to infinity? Write your response in your notes and give a reason why.

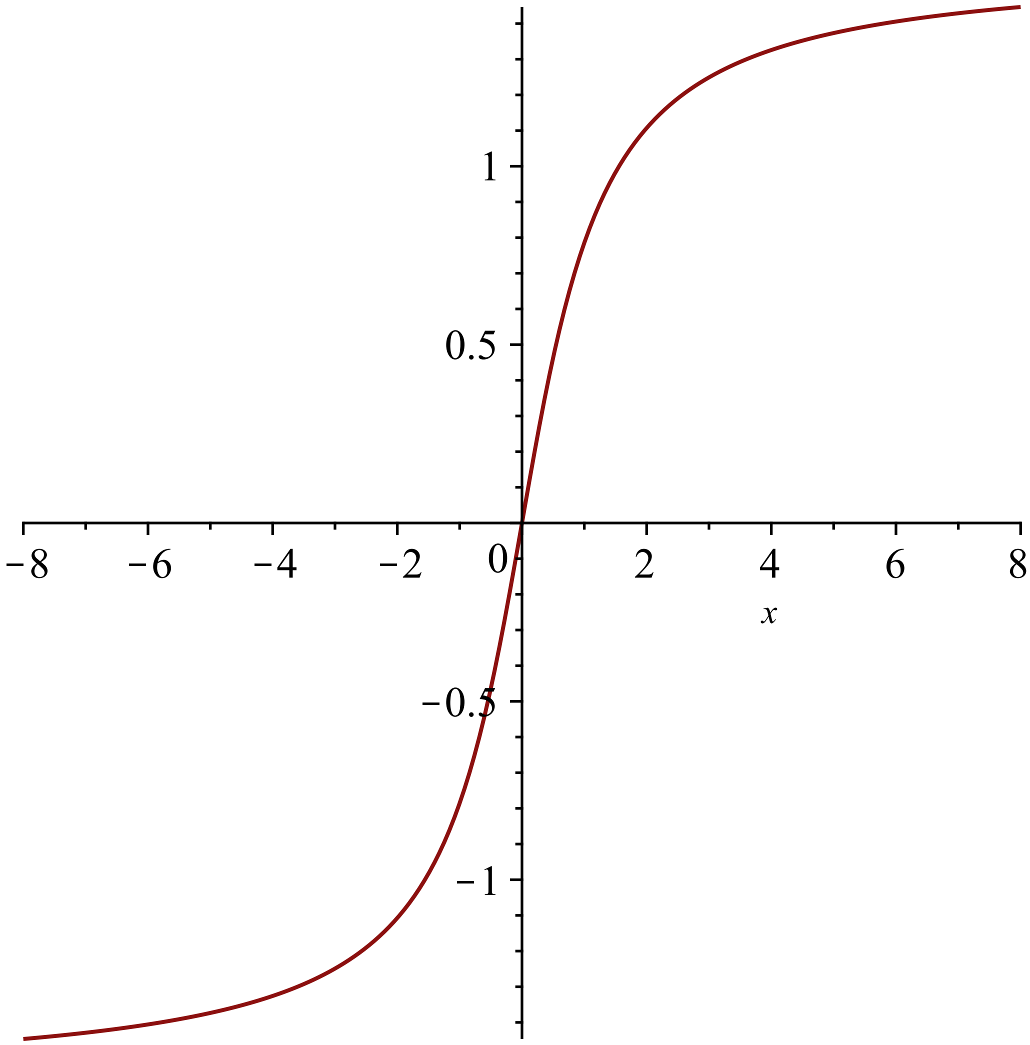
As $x$ gets large, its reciprocal gets small and so the limit is 0. Here is a plot where we can see the vertical asymptote at 0 and that the graph tends to 0 as $x$ gets large.

```
> plot([x^(-1)],x=0..100,color=[blue],legend=[typeset(x^(-1))],
  linestyle=[solid]);
```




Example 3: arctan and tan

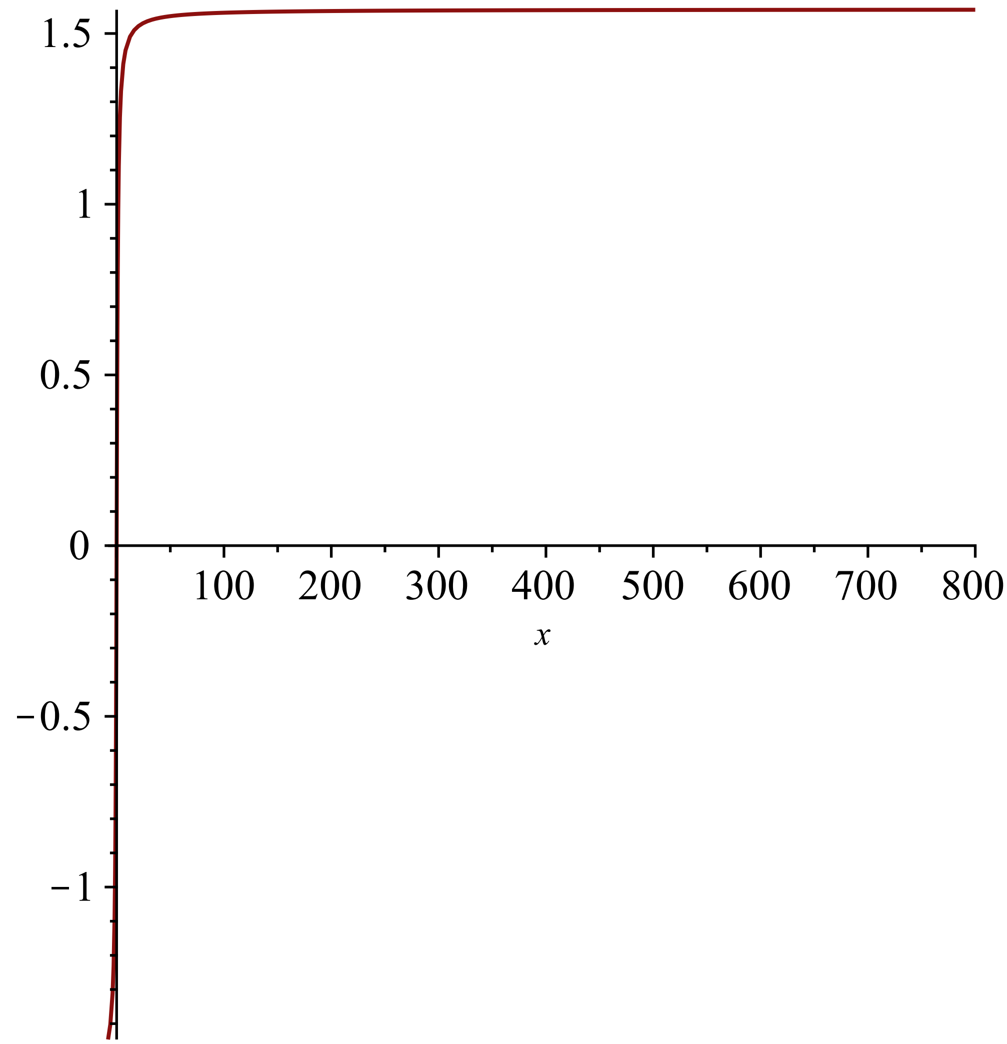
```
> plot(arctan(x),x=-8..8);
```




1) First roughly sketch this graph in your notes.

**<u>ACTIVITY</u>**: Modify the right endpoint of 8 below to really large numbers to showcase that arctan(x) does <u>NOT</u> continue to grow to infinity as x gets larger.

```
> plot(arctan(x),x=-8..800);
```



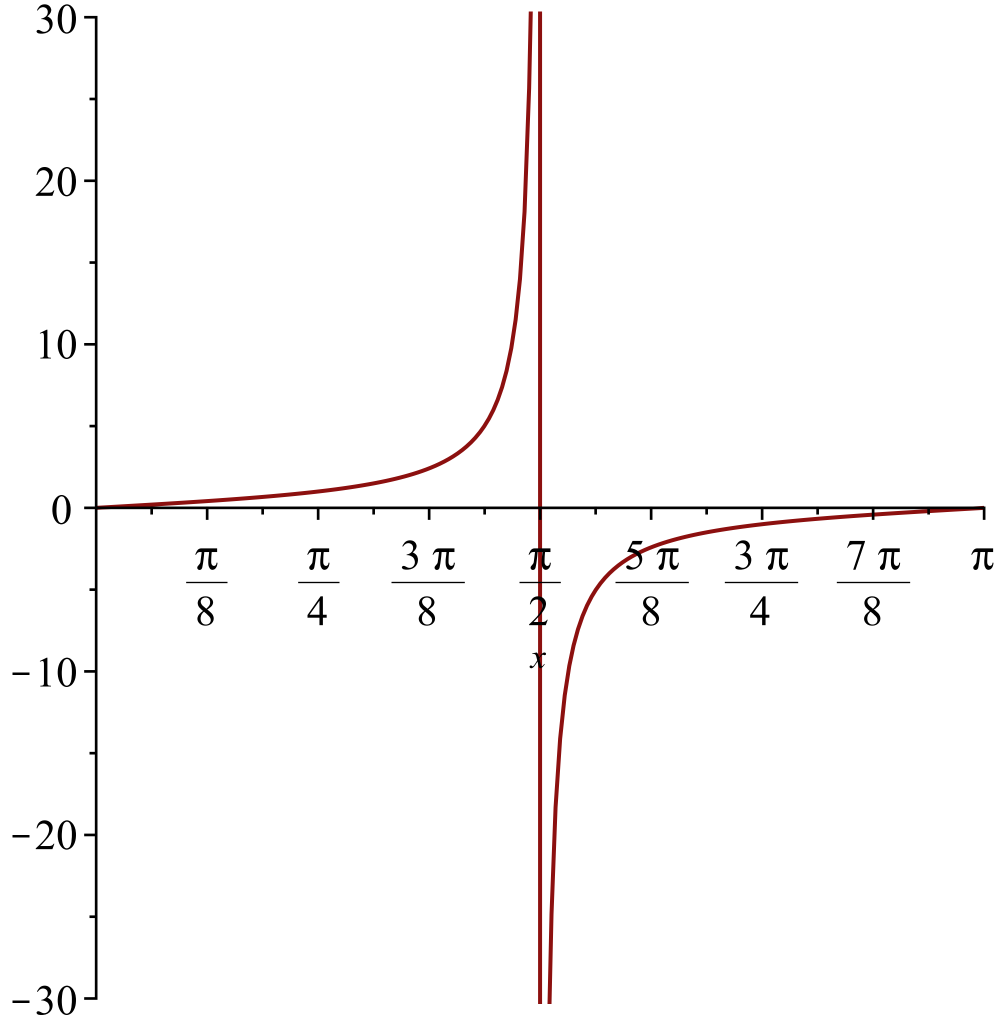

2) Read the following and then write the limit of arctan as *x* goes to infinity

arctan (x) is the inverse function of tan(x).
Note that Maple uses Pi in its commands so I'll use that notation here too.
Now tan(x) has an asymptote at Pi/2 and goes to positive infinity as *x* goes to Pi/2 from the left.
So by the inverse function, arctan(x) goes to Pi/2 from the left as x goes to infinity. You should have seen from the graph above that as *x* got really large there was an asymptote at about 1.57 radians, which is indeed Pi/2!

So the limit is Pi/2

3) Execute the graph and then write the limit of tan as *x* goes to Pi/2 from the right (from the <u>right</u>---not left!).

```
> plot(tan(x),x=0..Pi);
```



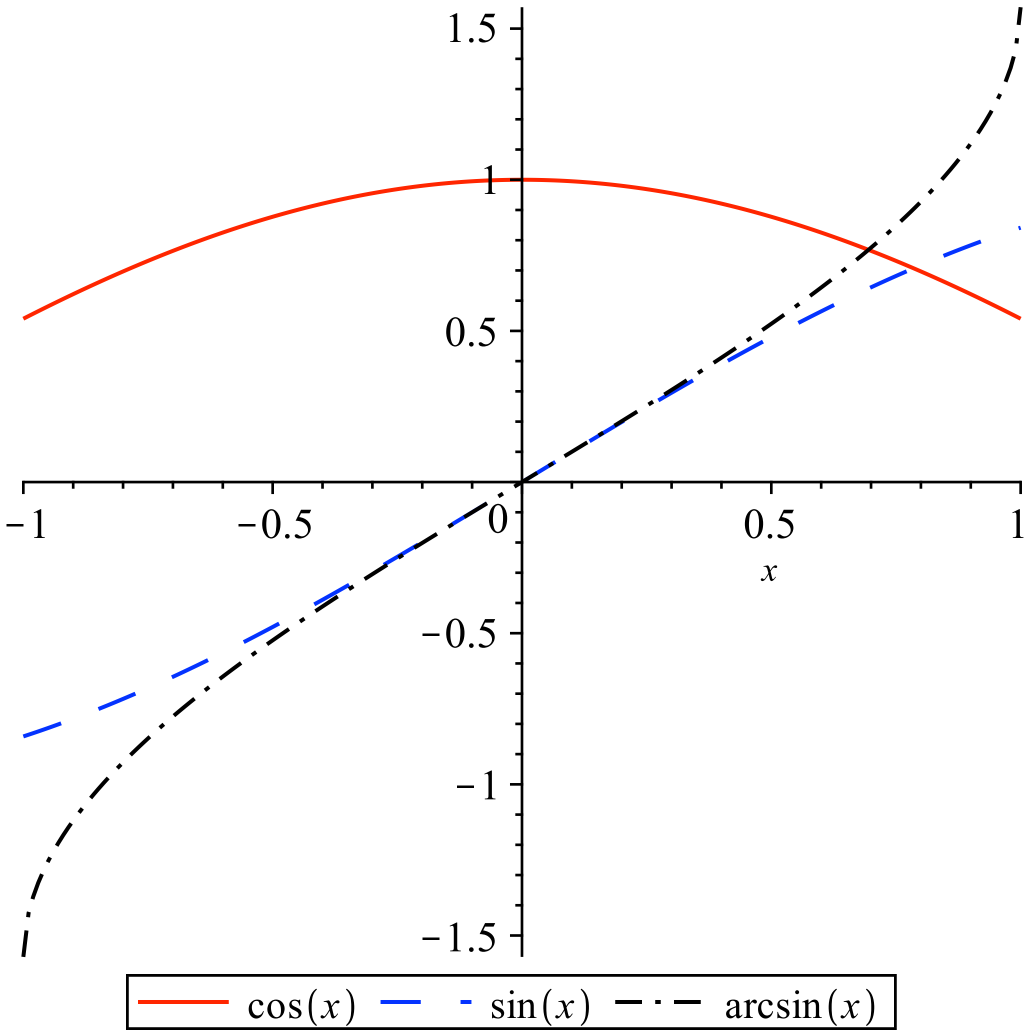

So as $x$ goes to Pi/2 from the right, tan goes to negative infinity. Hence the limit is $-\infty$.

4) What is the limit of tan as $x$ goes to Pi from the left?

The limit is 0. One can also see this from the definition of tangent as opposite over hypotenuse or sin over cos, as sin(Pi) goes to 0 in the numerator.

## Example 4: cos, sin, arcsin graphs

```
> plot([cos(x),sin(x),arcsin(x)],x=-1..1,color=[red,blue,black],
  legend=[typeset(cos(x)),typeset(sin(x)),typeset(arcsin(x))],
  linestyle=[solid,spacedash,dashdot]);
```




1) Roughly sketch these in your notes

2) What is the limit of arcsin(x) as x approaches 1 from the left side?

You can use the graph (the approximate y-value in radians should look familiar from above or reason from Example 3!) and/or reason from the inverse function perspective: sin(Pi/2) = 1 and so arcsin(1) = _____.

So the limit is Pi/2.