9.1 and 9.2 Group Work Target Practice

Given the following sequences (s_n) and series ($\sum$ or a finite sum), determine if they converge or diverge and EXPLAIN or SHOW WORK documenting why your answer is correct. If they converge, what value do they converge to (do NOT simplify)?

1. $s_n = \dfrac{n}{10} + \dfrac{10}{n}$.
   (a) Circle one: sequence
   (b) If a sequence: diverge
   (c) EXPLAIN or SHOW WORK
   $\dfrac{n}{10} + \dfrac{10}{n} = \dfrac{n^2+100}{10n}$
   Now $\lim_{n\to\infty} \frac{n^2+100}{10n} =$ L'Hôpital's rule $\lim_{n\to\infty} \frac{2n}{10}$ so $\lim_{n\to\infty} s_n = \infty$ and the sequence diverges.

2. Suppose the government proposes a tax cut totaling 100 million. We assume that all the people who have extra money spend 80% of it and save 20%. Thus, of the extra income generated by the tax cut, 100(.8) millon = 80 million is spent and becomes extra income to someone else. These people also spend 80% of their additional income, or 80(.8) million, and so on. Calculate the total additional spending created by such a tax cut.
   (a) Circle one: series
   If a series: Is it geometric? yes $a = 100 \cdot .8$ $x = .8$
   infinite series
   converge to 400 millon.
   (b) EXPLAIN or SHOW WORK This can be written as $\displaystyle\sum_{i=0}^{\infty} 100 \cdot .8(.8)^i$. Notice $|x| < 1$ because $x = .8$, so the series converges to $\frac{a}{1-x} = \frac{100 \cdot .8}{1-.8} = 400$ millon.

3. Once a day, eight tons of pollutants are dumped into a bay. Of this, 25% is removed by natural processes each day. What happens to the quantity of pollutants after 100 days?
   (a) series
   If a series: Is it geometric? yes $a = 8$ $x = .75$
   finite series
   converge to $\frac{8(1-.75^{100})}{1-.75} \approx 32$ tons of pullutants.
   (b) EXPLAIN or SHOW WORK This is a finite series so it automatically converges. To find out what it converges to, note that if 25% is removed, then 75% remains, so $x = .75$. The series can be written as $\displaystyle\sum_{i=0}^{99} 8(.75)^i$, which converges to $\frac{8(1-.75^{100})}{1-.75}$

4. $s_n = \frac{(-1)^n}{n}$
   (a) Circle one: sequence
   (b) If a sequence: converge to 0
   (c) EXPLAIN or SHOW WORK
   This is an alternating sequence, because the terms alternate in sign, but that doesn't stop the sequence from converging. The terms all approach 0 in the limit. The numerator is ± 1, while the denominator is larger and larger.