Test 2

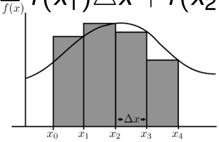
7.5, 8.1, 8.2, 8.4 (density only), 8.5 (work only), 9.1, 9.2 and 9.3
related test 1 material and material from prior classes

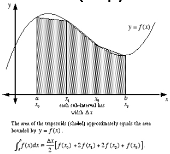
Local to Global Perspectives
Analyze small pieces to understand the big picture. Examples:

- numerical integration via rectangles
- area between two curves via rectangles
- volume by cylindrical disk or rectangular box slices
- total work via the work for each slice
  = force for each slice $\times$ displacement of that slice
- series diverges when sequence terms do not get smaller.
  (when they do get smaller anything can happen)

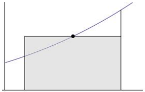
7.5 Numerical Methods

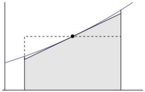
- Approximates integrals we can't evaluate directly, including discrete data
- n= number of intervals, $\triangle x = \frac{b-a}{n}$
- $Left(4) = f(x_0)\triangle x + f(x_1)\triangle x + f(x_2)\triangle x + f(x_3)\triangle x$ left endpoints
- $Right(4) = f(x_1)\triangle x + f(x_2)\triangle x + f(x_3)\triangle x + f(x_4)\triangle x$ right points





- $Trap(4) = \frac{Left(4)+Right(4)}{2}$
- $Mid(4) =$
  $f(\frac{x_0+x_1}{2})\triangle x + f(\frac{x_1+x_2}{2})\triangle x + f(\frac{x_2+x_3}{2})\triangle x + f(\frac{x_3+x_4}{2})\triangle x$ midpoints



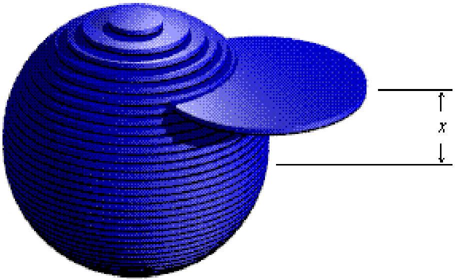


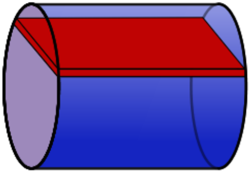
8.1 Area and Volume (Slice and Conquer)

- Area by slicing into rectangles with known length
- Volume by slicing into regions we know the area of
- Riemann sums with $\triangle x$ or $\triangle y \rightarrow \int_a^b dx$ or $\int_a^b dy$

$$\sum \pi(\tfrac{2}{5}y_i)^2 \triangle y \rightarrow \int_0^1 5(\frac{2}{5}y)^2 dy$$

What I want you to show me... picture, slice, Riemann sum, integral





8.1 Area Steps

1. Sketch a graph of the functions to find the enclosed region
2. Sketch a picture of a Riemann slice on your graph.
3. Base of the rectangle? Circle: Δx or Δy
4. Which function is larger in that variable (top for x, right for y)?
5. What is the height of the rectangle (top-bottom or right-left)?
6. What is the Riemann sum approximation? $\sum$ height $\cdot$ base $= \sum$
7. What is a and b (algebra finds the intersection points)?
8. Write the integral?

8.2 Volume Steps

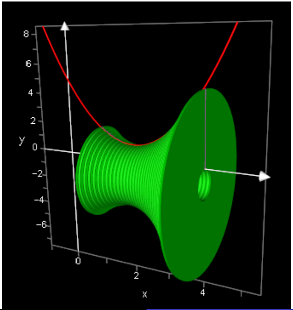
1. Sketch a graph of the object you want to find the volume of
2. Sketch a picture of a Riemann slice on your graph
3. What shape is it? Circle: rectangle (length·width·height) or cylinder/disk ($\pi \cdot$ radius^2·height)
4. Infinitesimal part of the slice? Circle: Δx or Δy
5. Sketch a diagram and show work to solve for any lengths you need
6. Circle any we used: Pythagorean theorem or similar triangles
7. What is the Riemann sum approximation? $\sum$
8. What is a and b?
9. Write the integral?

8.2 Volume (Revolutions) and Arc Length

- Volume by revolving a region about an axis
- Riemann sums with $\triangle x$ or $\triangle y \rightarrow \int_a^b dx$ or $\int_a^b dy$

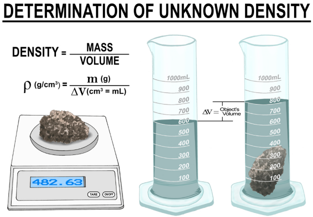
  $\sum \pi (\frac{2}{5} y_i)^2 \triangle y \rightarrow \int_0^1 5(\frac{2}{5} y)^2 dy$
- Common forms: $\displaystyle\int_a^b \pi r^2 dx$ and $\displaystyle\int_a^b \pi (r_{outer}^2 - r_{inner}^2) dx$
- Key is to figure out the radius (or radii) via pics

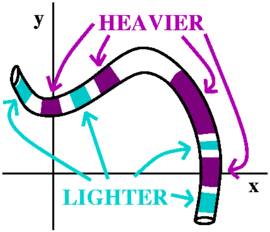
What I want you to show me... reasoning for radius, integral



8.4 Varying Density

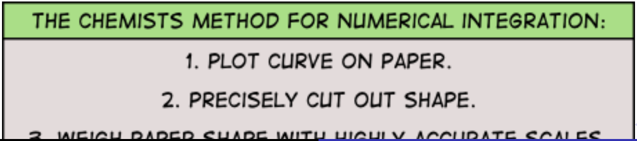
- Calc II density over length, area or volume, changing only in 1 dimension (Calc III for others)
- Slide/move so density is approximately constant:
  If $\delta = f(x)$, then slice $\perp x$
  If $\delta = f(r)$, then move from center outward via concentric circles/cylindrical shells $\int_a^b 2\pi r\delta(r)\, dr$
- Population: quantity per unit area or volume. People per square mile, bacteria per cc.
- Substance: mass per unit volume (gm per cc)





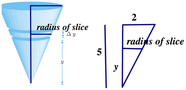
8.5 Work: Varying Force

- Work is force $\times$ distance
- Integrals apply when we vary the force, like Hook's Law to stretch (and hold) a spring, where $F(x) = kx$ and $W = \int F(x)dx$
- Sometimes need to calculate the force, like when it is a column of water:
  mass = density $\times$ volume
  F = mass $\times$ g
- Sometimes we don't need to multiply by g like when we have a density that already has a force component:
  weight (force) = volume $\times$62.4 lbs/ft^3
  Work = ($62.4 lb/ft^3$ $\times$ volume of slice) $\times$ displacement



Slicing for Volume, Density and Work Practice Sheet

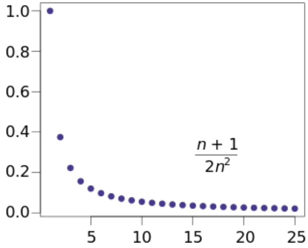
1. 

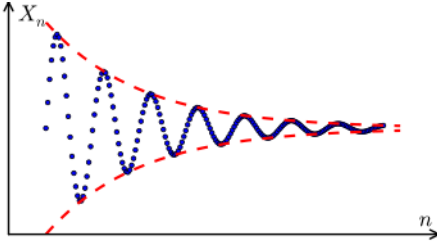
cylindrical disk volume $= \pi$ radius of slice$^2 \Delta y$
2. total cone volume: $\int_0^5 \pi (\frac{2y}{5})^2 dy$


Similar Δ: $\frac{\text{radius of slice}}{y} = \frac{2}{5}$ so $r = \frac{2y}{5}$
3. density $\delta(y)$ of the cone varies with it's height y:
mass = $\int_0^5 \delta(y)$ volume $= \int_0^5 \delta(y) \pi (\frac{2y}{5})^2 dy$
4. Work to pump the water out if cone filled to height of 4ft. = F d = ($62.5 lb/ft^3 \times$ volume) $\times$ d each slice displaced
$$= \int_0^4 (62.5 \times (\pi (\frac{2y}{5})^2 \times dy) \times (5 - y)$$

9.1 Sequences

- list of terms $s_1, s_2, ...s_n, ...$ often arranged in a fixed pattern
- algebraic, numeric and graphical representations
- new vocab: monotone, alternating, recursive, bounded
- $\lim_{n\to\infty} s_n$? converges or diverges?



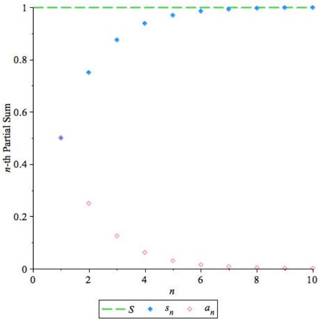


9.2 Series: Geometric

- ratio between any two consecutive terms is constant.

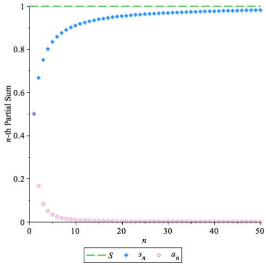
  sum of the first n terms: $\frac{a(1-x^n)}{1-x}$. Careful of # terms and starting index. $\lim_{n\to\infty} \frac{a(1-x^n)}{1-x} = \frac{a}{1-x}$ if $|x| < 1$

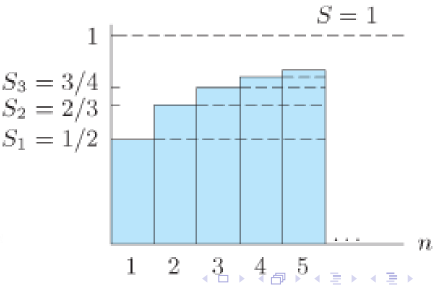
  Example: $\sum_{i=0}^{\infty} \frac{1}{2}\frac{1}{2}^{i} = \sum_{i=1}^{\infty} \frac{1}{2}^{i}$



9.3 Series: Partial Sums

- $\sum_{n=1}^{\infty} a_n$ and convergence? [9.3, 9.4, 9.5, chapter 10]
- n^{th} partial sum: $\sum_{i=1}^{n} a_i$ where a_i may not be geometric
  sequence of partial sums S_n converges $\Leftrightarrow$ series does
  so examine $\lim_{n\to\infty}$ n^{th} partial sums
- Example: $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ $\quad S_n = \frac{n}{n+1}$ $\quad \lim_{n\to\infty} S_n = 1$





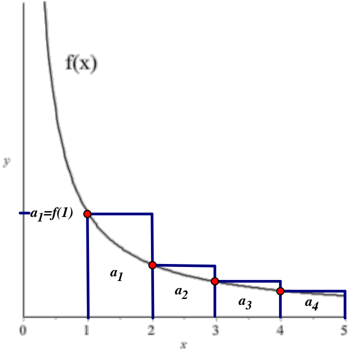
9.3: Limits and Linearity for Convergence or Divergence

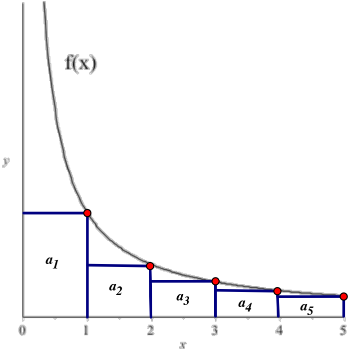
- terms not getting smaller: $\lim_{n\to\infty} a_n \neq 0$ or DNE, then partial sums diverge and so does the series. Example: $\sum_{n=1}^{\infty} \frac{5+n}{2n+1}$
- Linearity: $\sum_{n=1}^{\infty} a_n$ converges to S and $\sum_{n=1}^{\infty} b_n$ converges to T, and k is any constant, then $\sum_{n=1}^{\infty} ka_n + b_n$ converges to $kS + T$.

  Application 1: add two geometric series (converge)
  Application 2: add divergent & convergent series (diverge)
  Example: $\sum_{n=1}^{\infty} \frac{1}{2}^n + (-1)^n$. If convergent, then subtract convergent $\sum_{n=1}^{\infty} \frac{1}{2}^n$ and the result should converge.

9.3: Integral Test Bounds

If series has terms that are decreasing and positive, the integral test not only tells us about convergence, but also bounds the series:





$$\int_1^\infty f(x)dx \leq \sum a_n \leq a_1 + \int_1^\infty f(x)dx$$

9.2 Geometric Series versus 9.3 p-Series

- ratio between any two consecutive terms is constant.
  sum of the first n terms: $\dfrac{a(1-x^n)}{1-x}$. Careful of # terms and starting index. $\lim_{n\to\infty} \dfrac{a(1-x^n)}{1-x} = \dfrac{a}{1-x}$ if $|x| < 1$
- $\sum_{n=1}^{\infty} \dfrac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

$\sum_{n=1}^{\infty} (\frac{1}{2})^n = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \ldots$ geo series, $|x| = .5 < 1$ conv to $\dfrac{.5}{1-.5}$

$\sum_{n=1}^{\infty} \dfrac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} \ldots$ p series: $p = 2 > 1$ conv by integral test:
terms dec +:

$\int_1^{\infty} \frac{1}{x^2}\, dx = \lim_{b\to\infty} \int_1^b x^{-2}\, dx = \lim_{b\to\infty} \frac{x^{-1}}{-1}\Big|_1^b = 0 - -1$

$1 \leq \sum_{n=1}^{\infty} \frac{1}{n^2} \leq 1+$ first term = $1 + 1$

1. Is this a geometric series? yes no

   Geometric Series: $\sum_{i=0}^{\infty} ax^i$ where x is the common ratio and a is a constant. $\sum_{i=0}^{n} ax^i = \frac{a(1-x^{n+1})}{1-x}$.

   $\sum_{i=0}^{\infty} ax^i = \frac{a}{1-x}$ provided $|x| < 1$.

2. Can we apply the Terms not Getting Smaller? yes no

   Terms not Getting Smaller: For $\sum a_n$, if the $\lim_{n->\infty} a_n \neq 0$, then the infinite series does not converge.

3. Are the terms decreasing and positive eventually, and if so is this an integral we can do? yes no

   Integral Test: For $\sum a_n$, if the terms are decreasing and $a_n > 0$, then the series behaves the same way as $\int_a^{\infty} a_n dn$, & $\int_a^{\infty} f(x)dx \leq \sum a_n \leq$ 1st term $+ \int_a^{\infty} f(x)dx$.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IT PUSHES SOME OLD STUFF OUT OF MY BRAIN!

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