Question 1

Not complete

Points out of 4.00

Open
https://www.geogebra.org/m/thu5sxs3
Move the a slider around to actively engage with the impact of a scalar multiple $a = \lambda$ varying over the reals from -5..5 times the vector $\vec{u} = \begin{bmatrix} 1 \\ 4 \\ 7 \end{bmatrix}$, shown in the graph as $\lambda\vec{u}$. You can also turn the graph in 3-space to see different viewpoints. Drag the slider to see the scalar multiples and solidify the visualization.

Next, set the slider at $a = -.5$. How does $-.5\vec{u}$ compare to $\vec{u}$? $-.5\vec{u}$ is...

- ○ on the same line through the origin
- ○ on a different line through the origin

- ○ pointed in the same direction
- ○ pointed in a different direction

- ○ the same length
- ○ shorter
- ○ longer

Next, open
https://www.geogebra.org/3d/gkghsbuu
Move the $a, b,$ and c sliders around to modify the generic vector $\vec{u} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$ and actively engage with the parallelogram in 3-space formed by $\vec{u}$ and $\vec{v} = \begin{bmatrix} -2 \\ -3 \\ -4 \end{bmatrix}$ as the coordinates of $\vec{u}$ change. Also notice the connection of diagonal of the parallelogram to $\vec{u} + \vec{v}$. You can also turn the graph in 3-space to see different viewpoints. Drag the sliders to solidify the visualizations.

- ○ ready to proceed
- ○ ready to proceed

Check

Question **2**

Not complete

Points out of 1.00

Compute vectors a) $\vec{u}+\vec{v}$, the diagonal of the parallelogram, and then b) $\vec{u}-2\vec{v}$, where

$\vec{u}=\begin{bmatrix}-4\\4\end{bmatrix}$, $\vec{v}=\begin{bmatrix}-7\\-2\end{bmatrix}$

Part a) $\vec{u}+\vec{v}=$

Part b) $\vec{u}-2\vec{v}=$

Check

Question **3**

Not complete

Points out of 1.00

Write a system of equations that is equivalent to the given vector equation.

$$x_1 \begin{bmatrix} 3 \\ -2 \\ 8 \end{bmatrix} + x_2 \begin{bmatrix} 5 \\ 0 \\ -9 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 8 \end{bmatrix}$$

x_1 +

x_2=

x_1 +

x_2=

x_1 +

x_2=

Check

Question **4**

Not complete

Points out of 9.00

A mining company has two mines. One day's operation at the first mine produces ore that contains 30 metric tons of copper and 600 kilograms of silver, while one day's operation at the second mine produces ore that contains 40 metric tons of copper and 380 kilograms of silver. We represent this in vector form as $v_1 = \begin{bmatrix} 30 \\ 600 \end{bmatrix}$, where v_1 is the output per day at mine 1, and $v_2 = \begin{bmatrix} 40 \\ 380 \end{bmatrix}$, where v_2 is the output per day at mine 2.

Part a) What physical interpretation can be given to the vector $4v_1$?

- ○ the output of silver at mine 1 after 4 days of operation
- ○ the output at mine 1 after 4 days of operation
- ○ the output at mine 2 after 4 days of operation
- ○ the output at both mines
- ○ other

Part b) Suppose the company operates mine 1 for x_1 days and mine 2 for x_2 days. Write a vector equation, **with mine 1 first**, of the form $\_ \begin{bmatrix} \_ \\ \_ \end{bmatrix} + \_ \begin{bmatrix} \_ \\ \_ \end{bmatrix} = \begin{bmatrix} \_ \\ \_ \end{bmatrix}$ whose solution would give the number of days each mine should operate in order to produce 240 tons of copper and 2824 kilograms of silver (do not solve the system).

○ x_1 ○ x_2 ○ other	[ ]	+	○ x_1 ○ x_2 ○ other	[ ]	= [ ]
	[ ]			[ ]	[ ]

Check

Question **5**

Not complete

Points out of 1.00

Write a vector equation that is equivalent to the given homogeneous system of equations

$x_2 + 7x_3 = 0$

$2x_1 + 4x_2 - 5x_3 = 0$

$-8x_1 + 9x_2 - 5x_3 = 0$

$x_1 \begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix} + x_2 \begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix} + x_3 \begin{bmatrix} \_ \\ \_ \\ \_ \end{bmatrix} =$

	[ ]		[ ]		[ ]		[ ]
x_1	[ ]	$+x_2$	[ ]	$+x_3$	[ ]	$=$	[ ]
	[ ]		[ ]		[ ]		[ ]

Check

Question **6**

Not complete

Points out of 3.00

Open
https://www.geogebra.org/m/gyepfveh
and drag the weight controller point in the graph all around the plane to change the weights a and b and actively engage with the different linear combinations of $a \begin{bmatrix} 1 \\ 1 \end{bmatrix} + b \begin{bmatrix} -1 \\ -1 \end{bmatrix} = \begin{bmatrix} a \\ a \end{bmatrix} + \begin{bmatrix} -b \\ -b \end{bmatrix} = \begin{bmatrix} a-b \\ a-b \end{bmatrix}$. Continue until you solidify the visualization. What does the visualization suggest about the span of the two vectors?

- ◯ it is the line $y = x$
- ◯ it is the entire plane
- ◯ other

Next open
https://www.geogebra.org/3d/rp6n8wq2
and drag the sliders a, b, c to change the weights a, b, c and actively engage with the different linear combinations of $a \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + b \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} + c \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix}$ in 3-space. Next turn the figure so you can see all the vectors. Then turn it back so it is a "head on" view. Continue until you solidify the visualization. What does this suggest about the span of the three vectors?

- ◯ it is an infinite line
- ◯ it is an infinite plane
- ◯ it is all of 3-space
- ◯ other

Lastly, open
https://www.geogebra.org/m/rdsy89zj
and drag the sliders a, b, c to change the weights a, b, c and actively engage with the different linear combinations of $a \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} + b \begin{bmatrix} 4 \\ -1 \\ 6 \end{bmatrix} + c \begin{bmatrix} 7 \\ 8 \\ -1 \end{bmatrix}$ in 3-space. After using all three sliders, can you turn the vectors so that all the linear combination vectors are in the same plane, i.e. a "head on" view?

- ◯ yes, the span is an infinite plane
- ◯ no, the span is all of 3-space, an infinite volume
- ◯ other

Check

Question **7**

Not complete

Points out of 9.00

As Francis Su writes in *Mathematics for Human Flourishing*,

$$x\begin{bmatrix}1\\3\end{bmatrix}+y\begin{bmatrix}2\\-1\end{bmatrix}+z\begin{bmatrix}1\\5\end{bmatrix}=\begin{bmatrix}8\\16\end{bmatrix}$$

> This is now a geometric question: if you have a spaceship at the origin (0,0) in a two-dimensional plane, and it has three thrusters that can push you in the directions $\begin{bmatrix}1\\3\end{bmatrix}$, $\begin{bmatrix}2\\-1\end{bmatrix}$ and $\begin{bmatrix}1\\5\end{bmatrix}$, will some combination allow you to reach $\begin{bmatrix}8\\16\end{bmatrix}$? From that perspective, any two of these thrusters will suffice to move you about this plane, so the math explorer sees that there are multiple solutions... there's an infinite number. She might next think about the system differently...

Write a system of equations that is equivalent to Francis Su's vector equation.

[ ] x + [ ] y+ [ ] z= [ ]

[ ] x + [ ] y+ [ ] z= [ ]

Is this a valid argument for Su's reasoning of infinite solutions?

> We see from the coefficients of the variables that we have two planes that are not parallel nor overlapping, so they must intersect in a line of infinite solutions.

○ yes

○ no

Check

Question **8**

Not complete

Points out of 6.00

To help you make connections between vector equations, an augmented matrix, its row echelon form (Gaussian form), consistent systems, linear combinations, and span, drag and drop the tiles just below the table into the relevant portions of the table.

	$\vec{b}=\begin{bmatrix}7\\4\\-3\end{bmatrix}, \vec{a}_1=\begin{bmatrix}1\\-2\\-5\end{bmatrix}, \vec{a}_2=\begin{bmatrix}2\\5\\6\end{bmatrix}$	$\vec{b}=\begin{bmatrix}-3\\8\\1\end{bmatrix}, \vec{a}_1=\begin{bmatrix}1\\-2\\3\end{bmatrix}, \vec{a}_2=\begin{bmatrix}5\\-13\\-3\end{bmatrix}$
$c_1\vec{a}_1+c_2\vec{a}_2=\vec{b}$	$c_1\begin{bmatrix}1\\-2\\-5\end{bmatrix}+c_2\begin{bmatrix}2\\5\\6\end{bmatrix}=\begin{bmatrix}7\\4\\-3\end{bmatrix}$	$c_1\begin{bmatrix}1\\-2\\3\end{bmatrix}+c_2\begin{bmatrix}5\\-13\\-3\end{bmatrix}=\begin{bmatrix}-3\\8\\1\end{bmatrix}$
Augmented matrix for the previous vector equation	$\begin{bmatrix}1&2&7\\-2&5&4\\-5&6&-3\end{bmatrix}$	$\begin{bmatrix}1&5&-3\\-2&-13&8\\3&-3&1\end{bmatrix}$
Row echelon form of the previous augmented matrix	$\begin{bmatrix}1&0&3\\0&1&2\\0&0&0\end{bmatrix}$	$\begin{bmatrix}1&5&-3\\0&-3&2\\0&0&-2\end{bmatrix}$
Does the augmented matrix represent a consistent system?	☐	☐
Is $\vec{b}$ a linear combination of $\vec{a}_1$ and $\vec{a}_2$	☐	☐
Is $\vec{b}$ in the span of $\vec{a}_1$ and $\vec{a}_2$	☐	☐

Yes No

Check

Question **9**

Not complete

Points out of 2.00

Can a linear combination of vectors in $\mathbb{R}^2$ or $\mathbb{R}^3$ ever be 1?

- ◯ yes
- ◯ no

Is a linear combination of $\begin{bmatrix}1\\0\\0\end{bmatrix}$ and $\begin{bmatrix}0\\1\\0\end{bmatrix}$ inside a plane in $\mathbb{R}^2$ or $\mathbb{R}^3$?

- ◯ inside in plane in $\mathbb{R}^2$
- ◯ inside in plane in $\mathbb{R}^3$

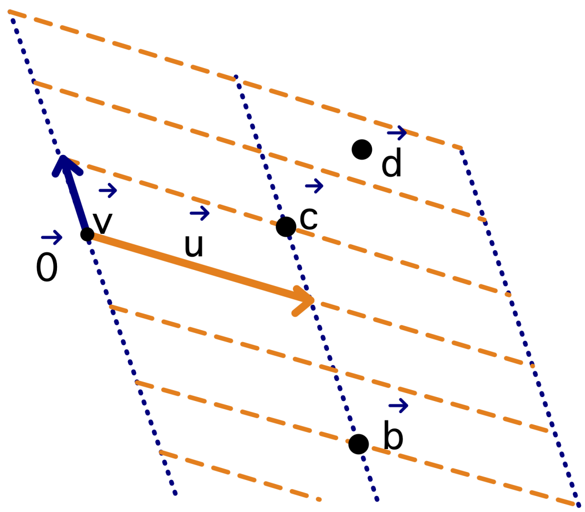
Check

Question **10**

Not complete

Points out of 6.00

The vectors referenced in this questions are those depicted in the figure below. For simplicity I've labeled the vectors $\vec{b}$, $\vec{c}$, and $\vec{d}$ at their tips; the tails are at the origin.



Write each as a linear combination of $\vec{u}$ and $\vec{v}$. If weights are not whole numbers, please use decimals.

$\vec{c}$ = [ ] $\vec{u}$ + [ ] $\vec{v}$

$\vec{d}$ = [ ] $\vec{u}$ + [ ] $\vec{v}$

$\vec{b}$ = [ ] $\vec{u}$ + [ ] $\vec{v}$

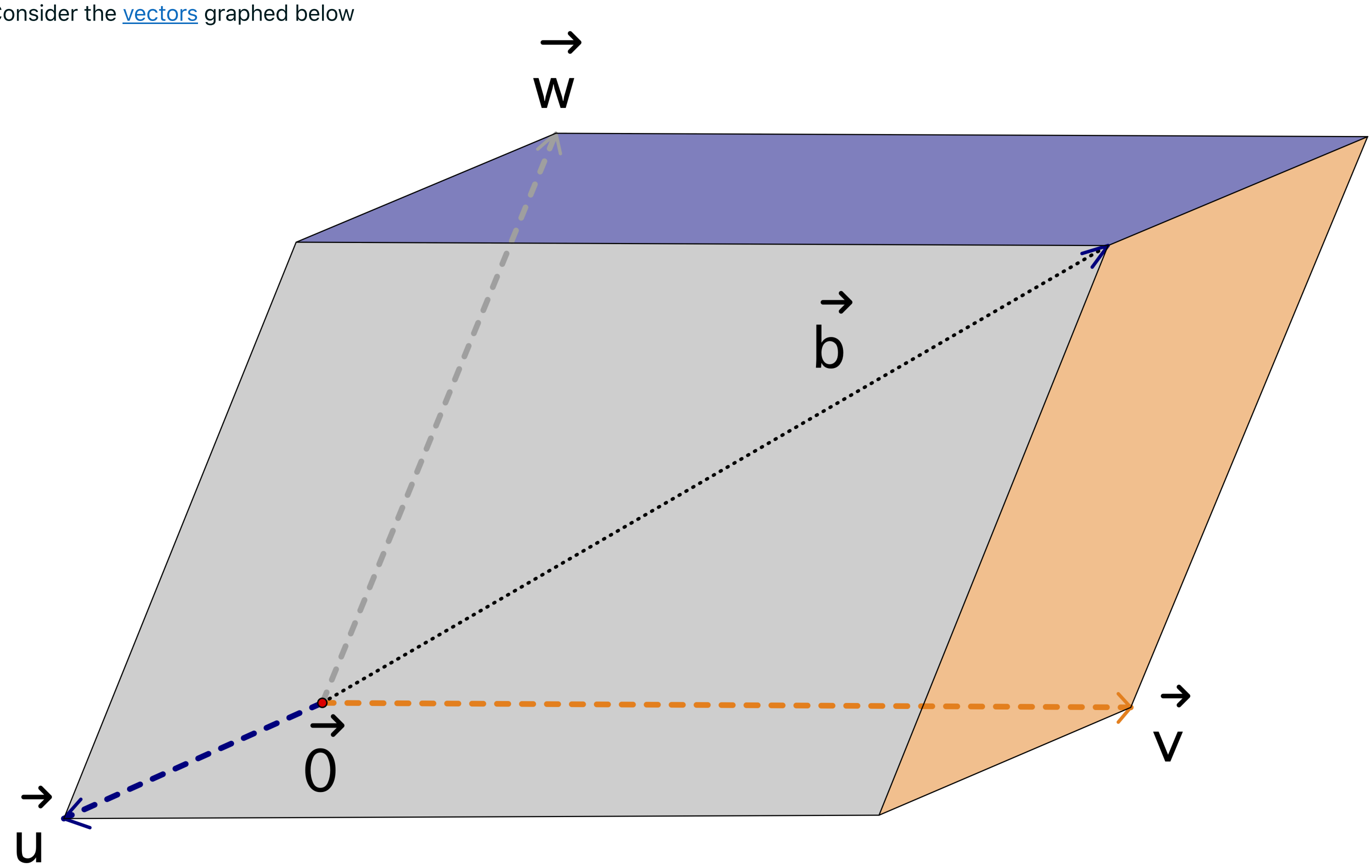
Check

Question **11**

Not complete

Points out of 4.00

Consider the vectors graphed below



part 1: Which of the following correctly expresses $\vec{b}$ as a linear combination of $\vec{u}$, $\vec{v}$, and $\vec{w}$

- ○ $\vec{b} = \vec{v} - \vec{u}$
- ○ $\vec{b} = \vec{w} + \vec{v}$
- ○ $\vec{b} = \vec{u} + \vec{v} + \vec{w}$

Part 2: Choose the correct description for each of the following:

$\vec{v} + \vec{u}$ points from the origin to

- ○ the top, front, left corner (where the blue and gray faces meet)
- ○ the bottom, front, right corner (where the gray and orange faces meet)
- ○ somewhere behind the figure
- ○ somewhere below the figure

$\vec{u} + \vec{w}$ points from the origin to

- ○ the top, front, left corner (where the blue and gray faces meet)
- ○ the bottom, front, right corner (where the gray and orange faces meet)
- ○ somewhere behind the figure
- ○ somewhere below the figure

$\vec{v} + \vec{w} - \vec{u}$ points from the origin to

- the top, front, left corner (where the blue and gray faces meet)
- the bottom, front, right corner (where the gray and orange faces meet)
- somewhere behind the figure
- somewhere below the figure

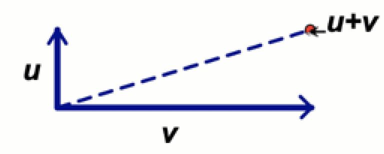
Check

Question **12**

Not complete

Points out of 2.00

Consider the span of two vectors that aren't on the same line. In the following movie, the vectors $\vec{u}$ and $\vec{v}$ are perpendicular to each other (orthogonal). Watch the animated gif through 2 cycles to solidify the visualization.



What does the animated gif show show?

○the span of two vectors that aren't on the same line consists of individual points on a lattice grid

○all in this plane are linear combinations of the two vectors but we can never leave this plane using only them.

Next imagine a collection of vectors that are not all in the same plane inside of $\mathbb{R}^3$. You can imagine axes or your fingers as long as they aren't all in the same plane. What is their span (all the linear combinations)?

○does not exist

○they smush into a plane

○the diagonal of a parallelepiped

○all of $\mathbb{R}^3$

Check

Question **13**

Not complete

Points out of 3.00

Describe the following geometrically inside of 3-space $\mathbb{R}^3$.

1. span$\left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} \right\}$ is

○a point in $\mathbb{R}^3$

○a line through the origin in $\mathbb{R}^3$

○a plane through the origin in $\mathbb{R}^3$

○all of 3-space $\mathbb{R}^3$

○other

2. span$\left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ -13 \\ -3 \end{bmatrix} \right\}$ is

○a point in $\mathbb{R}^3$

○a line through the origin in $\mathbb{R}^3$

○a plane through the origin in $\mathbb{R}^3$

○all of 3-space $\mathbb{R}^3$

○other

3. span$\left\{ \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix} \right\}$ is

○a point in $\mathbb{R}^3$

○a line through the origin in $\mathbb{R}^3$

○a plane through the origin in $\mathbb{R}^3$

○all of 3-space $\mathbb{R}^3$

○other

Check

Question **14**

Not complete

Points out of 1.00

Is the statement "The points in the plane corresponding to scalar multiples of $\begin{bmatrix} -5 \\ 2 \end{bmatrix}$ and scalar multiples of $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$ lie on a line through the origin." true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

- True and I found a phrase and page number from the text
- False and I can provide a counterexample
- other

Check

Question **15**

Not complete

Points out of 1.00

Is the statement "An example of a linear combination of vectors v_1 and v_2 is the vector $\frac{1}{2}v_1$" true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

- True and I found a phrase and page number from the text
- False and I can provide a counterexample
- other

Check

Question **16**

Not complete

Points out of 1.00

Is the statement "The set Span{u,v} is always visualized as a plane through the origin" true or false?

For true/false questions, the book instructs: if a statement is false, provide a specific counterexample. If it is true, quote a phrase and page number from the book.

- True and I found a phrase and page number from the text
- False and I can provide a counterexample
- other

Check

Question **17**

Not complete

Points out of 1.00

Which of the following are true?

◯one linear combination of vectors is the same as the span of a set of vectors

◯a linear combination of vectors and the span of a set of vectors are both ways of mixing vectors using addition and scalar multiplication

◯both of the above

◯none of the above

Check

Question **18**

Not complete

Points out of 1.00

We saw implicitplot in 1.1 and spacecurve in 1.3. Which is true?

◯there is no difference

◯implicitplot or implicitplot3d plots rows of an augmented matrix as equations representing lines or planes while spacecurve plots columns as vectors

◯spacecurve plots rows of an augmented matrix as equations representing lines or planes while implicitplot or implicitplot3d plots columns as vectors

Check

Question **19**

Not complete

Points out of 1.00

To solidify and prepare for upcoming work, review and contemplate your knowledge and any questions that remain as related to definitions, concepts, computations, and examples from 1.3, including

- algebra of vectors: coordinates, addition of vectors, scalar multiplication of vectors, properties like associativity under addition (property ii on p. 29), a linear combination with weights, zero vector, span of a set of vectors=all the linear combinations, is a vector in the span?, vector equation $\longrightarrow$ augmented matrix
- geometry of vectors in 2D and 3D: directed segment, parallelogram for addition, on same line for scalar multiplication of vectors, origin=zero vector, a linear combination geometrically in the plane or 3D, span=all the linear combinations geometrically in the plane or 3D, spaces of subsets of R^n spanned by vectors

Consider also 1.2, including

- matrix of a linear system: row echelon form (Gaussian), reduced row echelon form (Gauss-Jordan)
- pivots: pivot position of a matrix, pivot column of a matrix
- row reduction algorithm we will most commonly use: elimination by forward phase and back substitution to row echelon form
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions with free variables and parametric solutions

and 1.1, including:

- algebra of linear equations: coefficients and variables
- geometry of linear equations in 2D and 3D: lines and planes
- solution set: inconsistent: 0 solutions; consistent: 1 unique solution or infinite solutions
- matrix of a linear system: coefficient matrix, augmented matrix, triangular form
- row equivalent systems
- algorithm for solving a linear system using elementary row operations of replacement, interchange, and scaling

When you have finished reviewing and reflecting, select one of the following (both receive full credit)

○ I currently have no questions

○ I will continue solidifying and understand that help is available in Dr. Sarah's more extensive feedback that follows below each question after I finish and open back up an entire practice quiz (this is more extensive than the hints that I can access during the open quiz), in Dr. Sarah's glossary/Wiki which is embedded into ASULearn from the linked terms, in Dr. Sarah's office hours and forum, and in Math Lab and Tutoring

Check